Stoke Newington School
& Sixth Form

MEDIA ARTS & SCIENCE COLLEGE

Name	

A Level Mathematics

Summer Assignment

Maths A Level Summer Assignment & Transition Work

The summer assignment element should take no longer than **2 hours to complete**. Your summer assignment for each course must be submitted in the **relevant first** lesson in September. As a guideline, the transition work element will take 2 hours a week over the summer holiday.

This booklet has been produced to help you prepare for A-Level Maths. It is essential that you begin the course having maintained and developed your skills in the higher-level algebra content from the GCSE syllabus. You also need to understand the volume of work that will be required for A-level Maths. Completion of this booklet will provide evidence that you have the commitment and work ethic required. Additionally, this work will prepare you for the base line test in September.

INSTRUCTIONS: there are 2 parts to this booklet.

Part A: Summer Assignment (submitted in 1st lesson of the year)

This must be completed in full **on A4 paper** and handed in to your teacher during the first lesson of the year.

You must **show all workings** and set your work out clearly and logically, clearly labelling each question and sub-question. *Do not attempt to complete this on a printout of the assignment – there is not enough room for your full workings and solutions so it must be handed in on A4 paper.*

The assignment will be marked by your teacher. It should take **no more than 2 hours to complete**, once you have done any necessary revision in advance. The transition work in this booklet will prepare you for your summer assignment.

You will also be given a **1-hour baseline test** in the first lesson of the year.

Part B: Transition Work

This booklet has been produced by our exam board **Pearson Edexcel** to ensure that all A-Level Maths students across the country begin A level with the algebra skills necessary for the course. You are required to complete **all of this work over the summer**. As well as being compulsory, it will help with any necessary preparation for the summer assignment and the baseline test in the first lesson.

- Complete the answers to the exercises on A4 squared paper, with each topic and question number clearly labelled. You must show **all** working (final answers alone are not accepted).
- If you are stuck, use the Key Points and Examples to remind yourself of how to answer the questions.
- Answers to all of the questions are at the back of this booklet. You should use these answers to correct all of your work in a different coloured pen. If you have a different answer you are expected to make corrections, again showing all of your methods. Correcting your work is a **vital skill** in A-level mathematics as it contributes to improving your understanding in key areas

To summarise, in September you will be assessed in 3 ways:

1. Part A: Summer Assignment will be formally marked and graded by your teacher.
2. Part B: Transition Work should be self-marked and handed in to your teacher.
3. 1-hour Baseline Test in your first lesson, which will be formally marked and graded by your teacher. Parts A & B of this assignment will help you prepare for this.

Part A: Summer Assignment (2 hours work)

1) Show that $(x-3)^2(2x+5)$ simplifies to $ax^3 + bx^2 + cx + d$ where a, b, c and d are integers to be found.

(3)

2) Functions f and g are such that $f(x) = 4x - 5$ and $g(x) = \dfrac{12}{2x+3}$

(a) Find the value of g(-5).

(1)

(b) Find gf(x) in the simplest form.

(2)

© Find the inverse function $g^{-1}(x)$.

(2)

3) Sketch the graph of $f(x) = x^2 + 9x + 25$, writing the coordinates of the turning point and the coordinates of any intercepts with the coordinate axes.

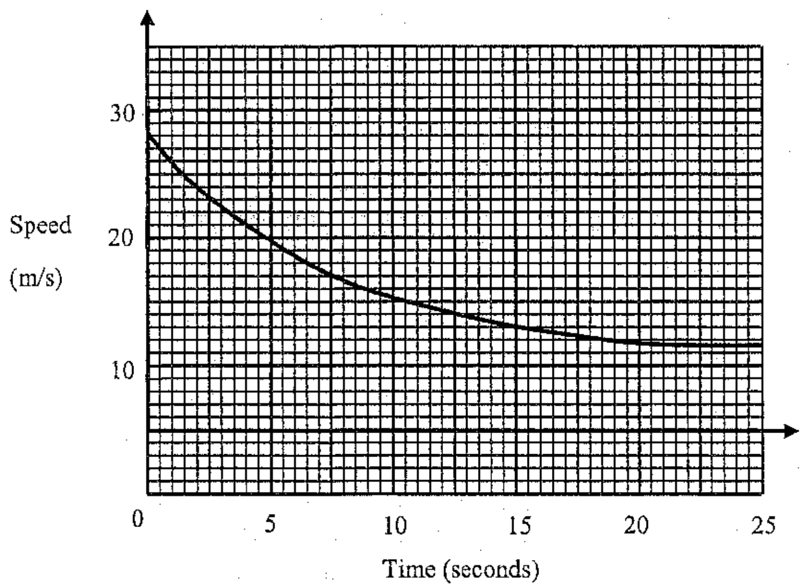
(3)

4) By completing the square, write the function $f(x) = 2x^2 - 3x - 8$ in the form $f(x) = a(x + b)^2 + c$

where a, b and c are rational numbers to be found.

(3)

5) The graph shows the speed, in metres per second, of a car as it approaches a set of traffic lights.



Use the graph to estimate the distance travelled by the car in the first 20 seconds.

(3)

6). A computer uses the iteration $x_{n+1} = \dfrac{3x_n^{\ 2} + 5}{9}$ to find one solution for a quadratic equation.

(a) What quadratic equation is being solved?

(1)

(b) Using this iterative formula, with $x_0 = 0$, find a solution to this quadratic equation to 3d.p.

(2)

7) Solve $\dfrac{x^2 - 16}{3} \geq 2x$

(3)

8). If a city's population grows by 20% every year, how many years will it take for this city to double in population?

(2)

9) The first 5 terms of a quadratic sequence are

-4 -5 -2 5 16

Find the expression, in terms of n, for the nth term of this sequence.

(3)

10) The following shows the results of a Higher Education Open Day survey of 100 people.

- 79 wanted to attend Maths.
- 55 wanted to attend Physics
- 32 wanted to attend Chemistry
- 43 wanted to attend both Maths and Physics
- 16 wanted to attend Physics and Chemistry
- 18 wanted to attend Maths and Chemistry
- 6 do not want to attend any of these subjects (they want to choose different subjects altogether)

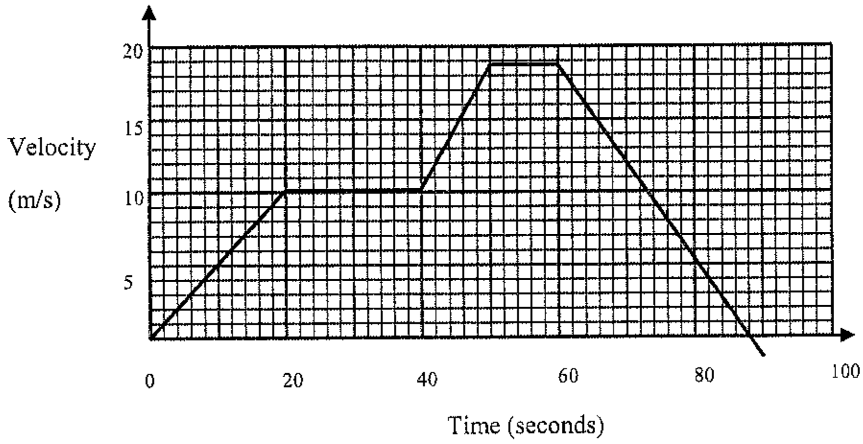
Find the probability that a randomly selected person from the survey wants to attend all three of these subjects.

(4)

11) Using the quadratic formula, find all the solutions to $2x^2 - x - 5 = 0$.

(2)

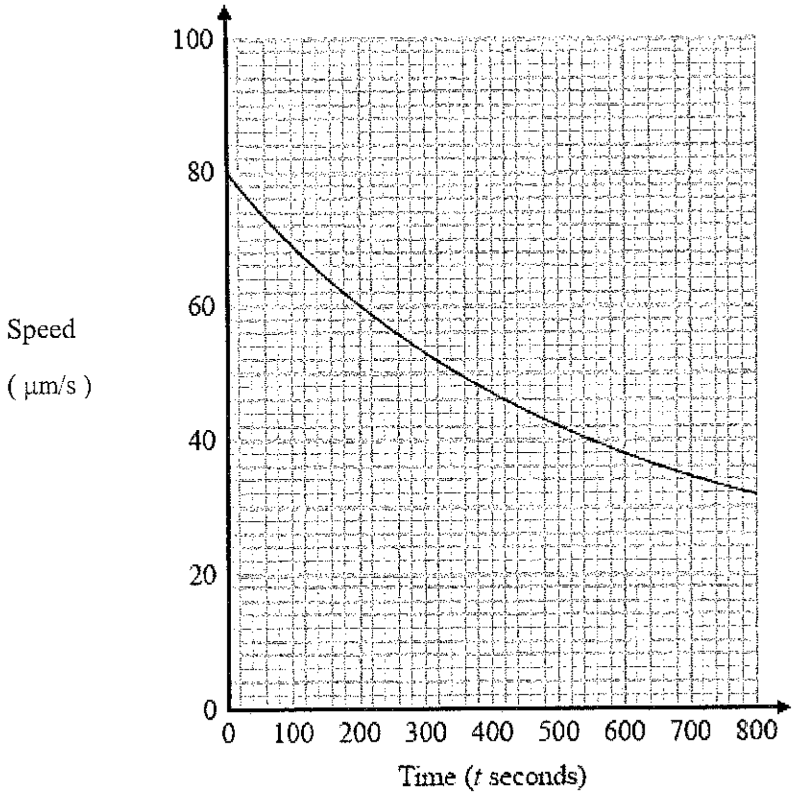
12). The graph shows the speed of a motorbike over a period of 1.5 minutes.



(a) Between which times was the acceleration the greatest? (1)

(b) Find the value of this acceleration. (2)

13) The graph shows the speed, in micrometres per second, of a snail crossing a garden path during a hot summer day.

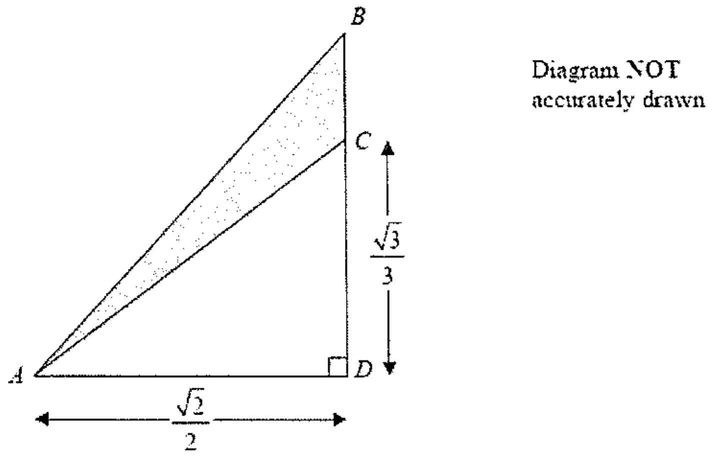


Work out the average deceleration of the snail in the first 600 seconds. (2)

14. Show that $\dfrac{1}{1+\dfrac{1}{\sqrt{2}}}$ can be written as $2 - \sqrt{2}$

(3)

15. *ABD* is a right angled triangle.



All measurements are given in centimetres.

C is the point on *BD* such that $CD = \dfrac{\sqrt{3}}{3}$

$AD = BD = \dfrac{\sqrt{2}}{2}$

Work out the exact area, in cm^2, of the shaded region.

.... cm^2

(3)

16. (a) Find the value of $\sqrt[3]{8 \times 10^6}$

....

(1)

(b) Find the value of $144^{\frac{1}{2}} \times 64^{-\frac{1}{3}}$

....

(2)

(c) Solve $3^{2x} = \dfrac{1}{81}$

$x =$ ....

(2)

17.

(a) Simplify $a^4 \times a^3$

....

(1)

(b) Simplify $(b^2)^7$

....

(1)

(c) Write down the value of 3^0

....

(1)

(d) Write down the value of 4^{-1}

....

(1)

Part B: Transition Work (Answers at end but you must show working)

Expanding brackets & simplifying expressions

A LEVEL LINKS
Scheme of work: 1a. Algebraic expressions – basic algebraic manipulation, indices and surds

Key points

- When you expand one set of brackets you must multiply everything inside the bracket by what is outside.
- When you expand two linear expressions, each with two terms of the form $ax + b$, where $a \neq 0$ and $b \neq 0$, you create four terms. Two of these can usually be simplified by collecting like terms.

Examples

Example 1 Expand $4(3x - 2)$

$4(3x - 2) = 12x - 8$	Multiply everything inside the bracket by the 4 outside the bracket

Example 2 Expand and simplify $3(x + 5) - 4(2x + 3)$

$3(x + 5) - 4(2x + 3)$ $= 3x + 15 - 8x - 12$	**1** Expand each set of brackets separately by multiplying $(x + 5)$ by 3 and $(2x + 3)$ by -4
$= 3 - 5x$	**2** Simplify by collecting like terms: $3x - 8x = -5x$ and $15 - 12 = 3$

Example 3 Expand and simplify $(x + 3)(x + 2)$

$(x + 3)(x + 2)$ $= x(x + 2) + 3(x + 2)$	**1** Expand the brackets by multiplying $(x + 2)$ by x and $(x + 2)$ by 3
$= x^2 + 2x + 3x + 6$ $= x^2 + 5x + 6$	**2** Simplify by collecting like terms: $2x + 3x = 5x$

Example 4 Expand and simplify $(x - 5)(2x + 3)$

$(x - 5)(2x + 3)$ $= x(2x + 3) - 5(2x + 3)$	**1** Expand the brackets by multiplying $(2x + 3)$ by x and $(2x + 3)$ by -5
$= 2x^2 + 3x - 10x - 15$ $= 2x^2 - 7x - 15$	**2** Simplify by collecting like terms: $3x - 10x = -7x$

Practice

1 Expand.

a $3(2x - 1)$ b $-2(5pq + 4q^2)$

c $-(3xy - 2y^2)$

> **Watch out!**
>
> When multiplying (or dividing) positive and negative numbers, if the signs are the same the answer is '+'; if the signs are different the answer is '–'.

2 Expand and simplify.

a $7(3x + 5) + 6(2x - 8)$ b $8(5p - 2) - 3(4p + 9)$

c $9(3s + 1) -5(6s - 10)$ d $2(4x - 3) - (3x + 5)$

3 Expand.

a $3x(4x + 8)$ b $4k(5k^2 - 12)$

c $-2h(6h^2 + 11h - 5)$ d $-3s(4s^2 - 7s + 2)$

4 Expand and simplify.

a $3(y^2 - 8) - 4(y^2 - 5)$ b $2x(x + 5) + 3x(x - 7)$

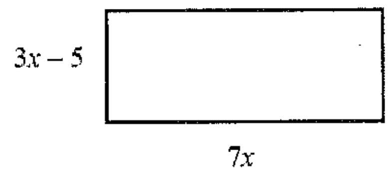
c $4p(2p - 1) - 3p(5p - 2)$ d $3b(4b - 3) - b(6b - 9)$

5 Expand $\frac{1}{2}(2y - 8)$

6 Expand and simplify.

a $13 - 2(m + 7)$ b $5p(p^2 + 6p) - 9p(2p - 3)$

7 The diagram shows a rectangle.
Write down an expression, in terms of x, for the area of the rectangle.
Show that the area of the rectangle can be written as $21x^2 - 35x$



8 Expand and simplify.

a $(x + 4)(x + 5)$ b $(x + 7)(x + 3)$

c $(x + 7)(x - 2)$ d $(x + 5)(x - 5)$

e $(2x + 3)(x - 1)$ f $(3x - 2)(2x + 1)$

g $(5x - 3)(2x - 5)$ h $(3x - 2)(7 + 4x)$

i $(3x + 4y)(5y + 6x)$ j $(x + 5)^2$

k $(2x - 7)^2$ l $(4x - 3y)^2$

Extend

9 Expand and simplify $(x + 3)^2 + (x - 4)^2$

10 Expand and simplify.

a $\left(x+\frac{1}{x}\right)\left(x-\frac{2}{x}\right)$ b $\left(x+\frac{1}{x}\right)^2$

11 Expand and simplify.

a $(x+4)(x+2)(x+1)$

b $(x+7)(x+1)(x-2)$

c $(x+7)(x-2)(x-9)$

d $(x-5)(x-4)(x-2)$

e $(2x+3)(x-1)(x-2)$

f $(3x-2)(2x+1)(3x-2)$

g $(x-3)^3$

h $(x-3)^4$

Surds and rationalising the denominator

A LEVEL LINKS
Scheme of work: 1a. Algebraic expressions – basic algebraic manipulation, indices and surds

Key points

- A surd is the square root of a number that is not a square number, for example $\sqrt{2}, \sqrt{3}, \sqrt{5},$ etc.
- Surds can be used to give the exact value for an answer.
- $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
- $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$
- To rationalise the denominator means to remove the surd from the denominator of a fraction.
- To rationalise $\frac{a}{\sqrt{b}}$ you multiply the numerator and denominator by the surd $\sqrt{b}$
- To rationalise $\frac{a}{b+\sqrt{c}}$ you multiply the numerator and denominator by $b-\sqrt{c}$

Examples

Example 1 Simplify $\sqrt{50}$

$\sqrt{50} = \sqrt{25 \times 2}$	**1** Choose two numbers that are factors of 50. One of the factors must be a square number
$= \sqrt{25} \times \sqrt{2}$	**2** Use the rule $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
$= 5 \times \sqrt{2}$	**3** Use $\sqrt{25} = 5$
$= 5\sqrt{2}$	

Example 2 Simplify $\sqrt{147} - 2\sqrt{12}$

$\sqrt{147} - 2\sqrt{12}$ $= \sqrt{49 \times 3} - 2\sqrt{4 \times 3}$	**1** Simplify $\sqrt{147}$ and $2\sqrt{12}$. Choose two numbers that are factors of 147 and two numbers that are factors of 12. One of each pair of factors must be a square number
$= \sqrt{49} \times \sqrt{3} - 2\sqrt{4} \times \sqrt{3}$	**2** Use the rule $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
$= 7 \times \sqrt{3} - 2 \times 2 \times \sqrt{3}$	**3** Use $\sqrt{49} = 7$ and $\sqrt{4} = 2$
$= 7\sqrt{3} - 4\sqrt{3}$ $= 3\sqrt{3}$	**4** Collect like terms

Example 3 Simplify $\left(\sqrt{7}+\sqrt{2}\right)\left(\sqrt{7}-\sqrt{2}\right)$

$\left(\sqrt{7}+\sqrt{2}\right)\left(\sqrt{7}-\sqrt{2}\right)$ $= \sqrt{49}-\sqrt{7}\sqrt{2}+\sqrt{2}\sqrt{7}-\sqrt{4}$ $= 7-2$ $= 5$	**1** Expand the brackets. A common mistake here is to write $\left(\sqrt{7}\right)^2 = 49$ **2** Collect like terms: $-\sqrt{7}\sqrt{2}+\sqrt{2}\sqrt{7}$ $= -\sqrt{7}\sqrt{2}+\sqrt{7}\sqrt{2} = 0$

Example 4 Rationalise $\frac{1}{\sqrt{3}}$

$\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$ $= \frac{1 \times \sqrt{3}}{\sqrt{9}}$ $= \frac{\sqrt{3}}{3}$	**1** Multiply the numerator and denominator by $\sqrt{3}$ **2** Use $\sqrt{9} = 3$

Example 5 Rationalise and simplify $\frac{\sqrt{2}}{\sqrt{12}}$

$\frac{\sqrt{2}}{\sqrt{12}} = \frac{\sqrt{2}}{\sqrt{12}} \times \frac{\sqrt{12}}{\sqrt{12}}$ $= \frac{\sqrt{2} \times \sqrt{4 \times 3}}{12}$ $= \frac{2\sqrt{2}\sqrt{3}}{12}$ $= \frac{\sqrt{2}\sqrt{3}}{6}$	**1** Multiply the numerator and denominator by $\sqrt{12}$ **2** Simplify $\sqrt{12}$ in the numerator. Choose two numbers that are factors of 12. One of the factors must be a square number **3** Use the rule $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$ **4** Use $\sqrt{4} = 2$ **5** Simplify the fraction: $\frac{2}{12}$ simplifies to $\frac{1}{6}$

Example 6 Rationalise and simplify $\frac{3}{2+\sqrt{5}}$

$\frac{3}{2+\sqrt{5}}=\frac{3}{2+\sqrt{5}}\times\frac{2-\sqrt{5}}{2-\sqrt{5}}$	**1** Multiply the numerator and denominator by $2-\sqrt{5}$
$=\frac{3(2-\sqrt{5})}{(2+\sqrt{5})(2-\sqrt{5})}$	**2** Expand the brackets
$=\frac{6-3\sqrt{5}}{4+2\sqrt{5}-2\sqrt{5}-5}$	**3** Simplify the fraction
$=\frac{6-3\sqrt{5}}{-1}$ $=3\sqrt{5}-6$	**4** Divide the numerator by -1 Remember to change the sign of all terms when dividing by -1

Practice

1 Simplify.

a $\sqrt{45}$ **b** $\sqrt{125}$

c $\sqrt{48}$ **d** $\sqrt{175}$

e $\sqrt{300}$ **f** $\sqrt{28}$

g $\sqrt{72}$ **h** $\sqrt{162}$

> **Hint**
>
> One of the two numbers you choose at the start must be a square number.

2 Simplify.

a $\sqrt{72}+\sqrt{162}$ **b** $\sqrt{45}-2\sqrt{5}$

c $\sqrt{50}-\sqrt{8}$ **d** $\sqrt{75}-\sqrt{48}$

e $2\sqrt{28}+\sqrt{28}$ **f** $2\sqrt{12}-\sqrt{12}+\sqrt{27}$

> **Watch out!**
>
> Check you have chosen the highest square number at the start.

3 Expand and simplify.

a $(\sqrt{2}+\sqrt{3})(\sqrt{2}-\sqrt{3})$ **b** $(3+\sqrt{3})(5-\sqrt{12})$

c $(4-\sqrt{5})(\sqrt{45}+2)$ **d** $(5+\sqrt{2})(6-\sqrt{8})$

4 Rationalise and simplify, if possible.

a $\frac{1}{\sqrt{5}}$ b $\frac{1}{\sqrt{11}}$

c $\frac{2}{\sqrt{7}}$ d $\frac{2}{\sqrt{8}}$

e $\frac{2}{\sqrt{2}}$ f $\frac{5}{\sqrt{5}}$

g $\frac{\sqrt{8}}{\sqrt{24}}$ h $\frac{\sqrt{5}}{\sqrt{45}}$

5 Rationalise and simplify.

a $\frac{1}{3-\sqrt{5}}$ b $\frac{2}{4+\sqrt{3}}$ c $\frac{6}{5-\sqrt{2}}$

Extend

6 Expand and simplify $\left(\sqrt{x}+\sqrt{y}\right)\left(\sqrt{x}-\sqrt{y}\right)$

7 Rationalise and simplify, if possible.

a $\frac{1}{\sqrt{9}-\sqrt{8}}$ b $\frac{1}{\sqrt{x}-\sqrt{y}}$

Rules of indices

A LEVEL LINKS
Scheme of work: 1a. Algebraic expressions – basic algebraic manipulation, indices and surds

Key points

- $a^m \times a^n = a^{m+n}$
- $\frac{a^m}{a^n} = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $a^0 = 1$
- $a^{\frac{1}{n}} = \sqrt[n]{a}$ i.e. the *n*th root of *a*
- $a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$
- $a^{-m} = \frac{1}{a^m}$
- The square root of a number produces two solutions, e.g. $\sqrt{16} = \pm 4$.

Examples

Example 1 Evaluate 10^0

$10^0 = 1$	Any value raised to the power of zero is equal to 1

Example 2 Evaluate $9^{\frac{1}{2}}$

$9^{\frac{1}{2}} = \sqrt{9}$ $= 3$	Use the rule $a^{\frac{1}{n}} = \sqrt[n]{a}$

Example 3 Evaluate $27^{\frac{2}{3}}$

$27^{\frac{2}{3}} = \left(\sqrt[3]{27}\right)^2$ $= 3^2$ $= 9$	**1** Use the rule $a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$ **2** Use $\sqrt[3]{27} = 3$

Example 4 Evaluate 4^{-2}

$4^{-2} = \frac{1}{4^2}$ $= \frac{1}{16}$	**1** Use the rule $a^{-m} = \frac{1}{a^m}$ **2** Use $4^2 = 16$

Example 5 Simplify $\frac{6x^5}{2x^2}$

$\frac{6x^5}{2x^2} = 3x^3$	$6 \div 2 = 3$ and use the rule $\frac{a^m}{a^n} = a^{m-n}$ to give $\frac{x^5}{x^2} = x^{5-2} = x^3$

Example 6 Simplify $\frac{x^3 \times x^5}{x^4}$

$\frac{x^3 \times x^5}{x^4} = \frac{x^{3+5}}{x^4} = \frac{x^8}{x^4}$ $= x^{8-4} = x^4$	**1** Use the rule $a^m \times a^n = a^{m+n}$ **2** Use the rule $\frac{a^m}{a^n} = a^{m-n}$

Example 7 Write $\frac{1}{3x}$ as a single power of x

$\frac{1}{3x} = \frac{1}{3}x^{-1}$	Use the rule $\frac{1}{a^m} = a^{-m}$, note that the fraction $\frac{1}{3}$ remains unchanged

Example 8 Write $\frac{4}{\sqrt{x}}$ as a single power of x

$\frac{4}{\sqrt{x}} = \frac{4}{x^{\frac{1}{2}}}$ $= 4x^{-\frac{1}{2}}$	**1** Use the rule $a^{\frac{1}{n}} = \sqrt[n]{a}$ **2** Use the rule $\frac{1}{a^m} = a^{-m}$

Practice

1 Evaluate.

a 14^0 b 3^0 c 5^0 d x^0

2 Evaluate.

a $49^{\frac{1}{2}}$ b $64^{\frac{1}{3}}$ c $125^{\frac{1}{3}}$ d $16^{\frac{1}{4}}$

3 Evaluate.

a $25^{\frac{3}{2}}$ b $8^{\frac{5}{3}}$ c $49^{\frac{3}{2}}$ d $16^{\frac{3}{4}}$

4 Evaluate.

a 5^{-2} b 4^{-3} c 2^{-5} d 6^{-2}

5 Simplify.

a $\dfrac{3x^2 \times x^3}{2x^2}$ b $\dfrac{10x^5}{2x^2 \times x}$

c $\dfrac{3x \times 2x^3}{2x^3}$ d $\dfrac{7x^3y^2}{14x^5y}$

e $\dfrac{y^2}{y^{\frac{1}{2}} \times y}$ f $\dfrac{c^{\frac{1}{2}}}{c^2 \times c^{\frac{3}{2}}}$

g $\dfrac{(2x^2)^3}{4x^0}$ h $\dfrac{x^{\frac{1}{2}} \times x^{\frac{3}{2}}}{x^{-2} \times x^3}$

Watch out!

Remember that any value raised to the power of zero is 1. This is the rule $a^0 = 1$.

6 Evaluate.

a $4^{-\frac{1}{2}}$ b $27^{-\frac{2}{3}}$ c $9^{-\frac{1}{2}} \times 2^3$

d $16^{\frac{1}{4}} \times 2^{-3}$ e $\left(\dfrac{9}{16}\right)^{-\frac{1}{2}}$ f $\left(\dfrac{27}{64}\right)^{-\frac{2}{3}}$

7 Write the following as a single power of x.

a $\dfrac{1}{x}$ b $\dfrac{1}{x^7}$ c $\sqrt[4]{x}$

d $\sqrt[5]{x^2}$ e $\dfrac{1}{\sqrt[3]{x}}$ f $\dfrac{1}{\sqrt[3]{x^2}}$

8 Write the following without negative or fractional powers.

a x^{-3} b x^{0} c $x^{\frac{1}{5}}$

d $x^{\frac{2}{5}}$ e $x^{-\frac{1}{2}}$ f $x^{-\frac{3}{4}}$

9 Write the following in the form ax^n.

a $5\sqrt{x}$ b $\dfrac{2}{x^3}$ c $\dfrac{1}{3x^4}$

d $\dfrac{2}{\sqrt{x}}$ e $\dfrac{4}{\sqrt[3]{x}}$ f 3

Extend

10 Write as sums of powers of x.

a $\dfrac{x^5+1}{x^2}$ b $x^2\left(x+\dfrac{1}{x}\right)$ c $x^{-4}\left(x^2+\dfrac{1}{x^3}\right)$

Factorising expressions

A LEVEL LINKS
Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

- Factorising an expression is the opposite of expanding the brackets.
- A quadratic expression is in the form $ax^2 + bx + c$, where $a \neq 0$.
- To factorise a quadratic equation find two numbers whose sum is b and whose product is ac.
- An expression in the form $x^2 - y^2$ is called the difference of two squares. It factorises to $(x - y)(x + y)$.

Examples

Example 1 Factorise $15x^2y^3 + 9x^4y$

$15x^2y^3 + 9x^4y = 3x^2y(5y^2 + 3x^2)$	The highest common factor is $3x^2y$. So take $3x^2y$ outside the brackets and then divide each term by $3x^2y$ to find the terms in the brackets

Example 2 Factorise $4x^2 - 25y^2$

$4x^2 - 25y^2 = (2x + 5y)(2x - 5y)$	This is the difference of two squares as the two terms can be written as $(2x)^2$ and $(5y)^2$

Example 3 Factorise $x^2 + 3x - 10$

$b = 3$, $ac = -10$	**1** Work out the two factors of $ac = -10$ which add to give $b = 3$ (5 and -2)
So $x^2 + 3x - 10 = x^2 + 5x - 2x - 10$	**2** Rewrite the b term ($3x$) using these two factors
$= x(x + 5) - 2(x + 5)$	**3** Factorise the first two terms and the last two terms
$= (x + 5)(x - 2)$	**4** $(x + 5)$ is a factor of both terms

Example 4 Factorise $6x^2 - 11x - 10$

$b = -11$, $ac = -60$ So $6x^2 - 11x - 10 = 6x^2 - 15x + 4x - 10$ $= 3x(2x - 5) + 2(2x - 5)$ $= (2x - 5)(3x + 2)$	**1** Work out the two factors of $ac = -60$ which add to give $b = -11$ (-15 and 4) **2** Rewrite the b term ($-11x$) using these two factors **3** Factorise the first two terms and the last two terms **4** $(2x - 5)$ is a factor of both terms

Example 5 Simplify $\dfrac{x^2-4x-21}{2x^2+9x+9}$

$\dfrac{x^2-4x-21}{2x^2+9x+9}$	**1** Factorise the numerator and the denominator
For the numerator: $b = -4$, $ac = -21$	**2** Work out the two factors of $ac = -21$ which add to give $b = -4$ (-7 and 3)
So $x^2 - 4x - 21 = x^2 - 7x + 3x - 21$ $= x(x - 7) + 3(x - 7)$ $= (x - 7)(x + 3)$	**3** Rewrite the b term ($-4x$) using these two factors **4** Factorise the first two terms and the last two terms **5** $(x - 7)$ is a factor of both terms
For the denominator: $b = 9$, $ac = 18$	**6** Work out the two factors of $ac = 18$ which add to give $b = 9$ (6 and 3)
So $2x^2 + 9x + 9 = 2x^2 + 6x + 3x + 9$ $= 2x(x + 3) + 3(x + 3)$ $= (x + 3)(2x + 3)$	**7** Rewrite the b term ($9x$) using these two factors **8** Factorise the first two terms and the last two terms **9** $(x + 3)$ is a factor of both terms
So $\dfrac{x^2-4x-21}{2x^2+9x+9} = \dfrac{(x-7)(x+3)}{(x+3)(2x+3)}$ $= \dfrac{x-7}{2x+3}$	**10** $(x + 3)$ is a factor of both the numerator and denominator so cancels out as a value divided by itself is 1

Practice

1 Factorise.

a $6x^4y^3 - 10x^3y^4$
b $21a^3b^5 + 35a^5b^2$
c $25x^2y^2 - 10x^3y^2 + 15x^2y^3$

> **Hint**
>
> Take the highest common factor outside the bracket.

2 Factorise

a $x^2 + 7x + 12$
b $x^2 + 5x - 14$
c $x^2 - 11x + 30$
d $x^2 - 5x - 24$
e $x^2 - 7x - 18$
f $x^2 + x - 20$
g $x^2 - 3x - 40$
h $x^2 + 3x - 28$

3 Factorise

a $36x^2 - 49y^2$
b $4x^2 - 81y^2$
c $18a^2 - 200b^2c^2$

4 Factorise

a $2x^2 + x - 3$
b $6x^2 + 17x + 5$
c $2x^2 + 7x + 3$
d $9x^2 - 15x + 4$
e $10x^2 + 21x + 9$
f $12x^2 - 38x + 20$

5 Simplify the algebraic fractions.

a $\dfrac{2x^2 + 4x}{x^2 - x}$
b $\dfrac{x^2 + 3x}{x^2 + 2x - 3}$
c $\dfrac{x^2 - 2x - 8}{x^2 - 4x}$
d $\dfrac{x^2 - 5x}{x^2 - 25}$
e $\dfrac{x^2 - x - 12}{x^2 - 4x}$
f $\dfrac{2x^2 + 14x}{2x^2 + 4x - 70}$

6 Simplify

a $\dfrac{9x^2 - 16}{3x^2 + 17x - 28}$
b $\dfrac{2x^2 - 7x - 15}{3x^2 - 17x + 10}$
c $\dfrac{4 - 25x^2}{10x^2 - 11x - 6}$
d $\dfrac{6x^2 - x - 1}{2x^2 + 7x - 4}$

Extend

7 Simplify $\sqrt{x^2 + 10x + 25}$

8 Simplify $\dfrac{(x+2)^2 + 3(x+2)^2}{x^2 - 4}$

Completing the square

A LEVEL LINKS
Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

- Completing the square for a quadratic rearranges $ax^2 + bx + c$ into the form $p(x + q)^2 + r$
- If $a \neq 1$, then factorise using a as a common factor.

Examples

Example 1 Complete the square for the quadratic expression $x^2 + 6x - 2$

$x^2 + 6x - 2$ $= (x + 3)^2 - 9 - 2$ $= (x + 3)^2 - 11$	**1** Write $x^2 + bx + c$ in the form $\left(x+\frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c$ **2** Simplify

Example 2 Write $2x^2 - 5x + 1$ in the form $p(x + q)^2 + r$

$2x^2 - 5x + 1$	**1** Before completing the square write $ax^2 + bx + c$ in the form $a\left(x^2+\frac{b}{a}x\right)+c$
$= 2\left(x^2-\frac{5}{2}x\right)+1$	**2** Now complete the square by writing $x^2-\frac{5}{2}x$ in the form $\left(x+\frac{b}{2}\right)^2-\left(\frac{b}{2}\right)^2$
$= 2\left[\left(x-\frac{5}{4}\right)^2-\left(\frac{5}{4}\right)^2\right]+1$	
$= 2\left(x-\frac{5}{4}\right)^2-\frac{25}{8}+1$	**3** Expand the square brackets – don't forget to multiply $\left(\frac{5}{4}\right)^2$ by the factor of 2
$= 2\left(x-\frac{5}{4}\right)^2-\frac{17}{8}$	**4** Simplify

Practice

1 Write the following quadratic expressions in the form $(x + p)^2 + q$

a $x^2 + 4x + 3$ b $x^2 - 10x - 3$

c $x^2 - 8x$ d $x^2 + 6x$

e $x^2 - 2x + 7$ f $x^2 + 3x - 2$

2 Write the following quadratic expressions in the form $p(x + q)^2 + r$

a $2x^2 - 8x - 16$ b $4x^2 - 8x - 16$

c $3x^2 + 12x - 9$ d $2x^2 + 6x - 8$

3 Complete the square.

a $2x^2 + 3x + 6$ b $3x^2 - 2x$

c $5x^2 + 3x$ d $3x^2 + 5x + 3$

Extend

4 Write $(25x^2 + 30x + 12)$ in the form $(ax + b)^2 + c$.

Solving quadratic equations by factorisation

A LEVEL LINKS
Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

- A quadratic equation is an equation in the form $ax^2 + bx + c = 0$ where $a \neq 0$.
- To factorise a quadratic equation find two numbers whose sum is b and whose products is ac.
- When the product of two numbers is 0, then at least one of the numbers must be 0.
- If a quadratic can be solved it will have two solutions (these may be equal).

Examples

Example 1 Solve $5x^2 = 15x$

$5x^2 = 15x$ $5x^2 - 15x = 0$	**1** Rearrange the equation so that all of the terms are on one side of the equation and it is equal to zero. Do not divide both sides by x as this would lose the solution $x = 0$.
$5x(x - 3) = 0$	**2** Factorise the quadratic equation. $5x$ is a common factor.
So $5x = 0$ or $(x - 3) = 0$	**3** When two values multiply to make zero, at least one of the values must be zero.
Therefore $x = 0$ or $x = 3$	**4** Solve these two equations.

Example 2 Solve $x^2 + 7x + 12 = 0$

$x^2 + 7x + 12 = 0$ $b = 7$, $ac = 12$	**1** Factorise the quadratic equation. Work out the two factors of $ac = 12$ which add to give you $b = 7$. (4 and 3)
$x^2 + 4x + 3x + 12 = 0$	**2** Rewrite the b term ($7x$) using these two factors.
$x(x + 4) + 3(x + 4) = 0$	**3** Factorise the first two terms and the last two terms.
$(x + 4)(x + 3) = 0$	**4** $(x + 4)$ is a factor of both terms.
So $(x + 4) = 0$ or $(x + 3) = 0$	**5** When two values multiply to make zero, at least one of the values must be zero.
Therefore $x = -4$ or $x = -3$	**6** Solve these two equations.

Example 3 Solve $9x^2 - 16 = 0$

$9x^2 - 16 = 0$ $(3x + 4)(3x - 4) = 0$ So $(3x + 4) = 0$ or $(3x - 4) = 0$ $x = -\frac{4}{3}$ or $x = \frac{4}{3}$	**1** Factorise the quadratic equation. This is the difference of two squares as the two terms are $(3x)^2$ and $(4)^2$. **2** When two values multiply to make zero, at least one of the values must be zero. **3** Solve these two equations.

Example 4 Solve $2x^2 - 5x - 12 = 0$

$b = -5$, $ac = -24$ So $2x^2 - 8x + 3x - 12 = 0$ $2x(x - 4) + 3(x - 4) = 0$ $(x - 4)(2x + 3) = 0$ So $(x - 4) = 0$ or $(2x + 3) = 0$ $x = 4$ or $x = -\frac{3}{2}$	**1** Factorise the quadratic equation. Work out the two factors of $ac = -24$ which add to give you $b = -5$. (-8 and 3) **2** Rewrite the b term ($-5x$) using these two factors. **3** Factorise the first two terms and the last two terms. **4** $(x - 4)$ is a factor of both terms. **5** When two values multiply to make zero, at least one of the values must be zero. **6** Solve these two equations.

Practice

1 Solve

a $6x^2 + 4x = 0$ **b** $28x^2 - 21x = 0$

c $x^2 + 7x + 10 = 0$ **d** $x^2 - 5x + 6 = 0$

e $x^2 - 3x - 4 = 0$ **f** $x^2 + 3x - 10 = 0$

g $x^2 - 10x + 24 = 0$ **h** $x^2 - 36 = 0$

i $x^2 + 3x - 28 = 0$ **j** $x^2 - 6x + 9 = 0$

k $2x^2 - 7x - 4 = 0$ **l** $3x^2 - 13x - 10 = 0$

2 Solve

a $x^2 - 3x = 10$ **b** $x^2 - 3 = 2x$

c $x^2 + 5x = 24$ **d** $x^2 - 42 = x$

e $x(x + 2) = 2x + 25$ **f** $x^2 - 30 = 3x - 2$

g $x(3x + 1) = x^2 + 15$ **h** $3x(x - 1) = 2(x + 1)$

Hint

Get all terms onto one side of the equation.

Solving quadratic equations by completing the square

A LEVEL LINKS
Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

- Completing the square lets you write a quadratic equation in the form $p(x + q)^2 + r = 0$.

Examples

Example 5 Solve $x^2 + 6x + 4 = 0$. Give your solutions in surd form.

$x^2 + 6x + 4 = 0$ $(x + 3)^2 - 9 + 4 = 0$ $(x + 3)^2 - 5 = 0$ $(x + 3)^2 = 5$ $x + 3 = \pm\sqrt{5}$ $x = \pm\sqrt{5} - 3$ So $x = -\sqrt{5} - 3$ or $x = \sqrt{5} - 3$	**1** Write $x^2 + bx + c = 0$ in the form $\left(x+\frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c = 0$ **2** Simplify. **3** Rearrange the equation to work out x. First, add 5 to both sides. **4** Square root both sides. Remember that the square root of a value gives two answers. **5** Subtract 3 from both sides to solve the equation. **6** Write down both solutions.

Example 6 Solve $2x^2 - 7x + 4 = 0$. Give your solutions in surd form.

$2x^2 - 7x + 4 = 0$ $2\left(x^2 - \frac{7}{2}x\right) + 4 = 0$ $2\left[\left(x - \frac{7}{4}\right)^2 - \left(\frac{7}{4}\right)^2\right] + 4 = 0$ $2\left(x - \frac{7}{4}\right)^2 - \frac{49}{8} + 4 = 0$ $2\left(x - \frac{7}{4}\right)^2 - \frac{17}{8} = 0$	**1** Before completing the square write $ax^2 + bx + c$ in the form $a\left(x^2 + \frac{b}{a}x\right) + c$ **2** Now complete the square by writing $x^2 - \frac{7}{2}x$ in the form $\left(x + \frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2$ **3** Expand the square brackets. **4** Simplify. *(continued on next page)* **5** Rearrange the equation to work out

$2\left(x-\frac{7}{4}\right)^2=\frac{17}{8}$	x. First, add $\frac{17}{8}$ to both sides.
$\left(x-\frac{7}{4}\right)^2=\frac{17}{16}$	**6** Divide both sides by 2.
$x-\frac{7}{4}=\pm\frac{\sqrt{17}}{4}$	**7** Square root both sides. Remember that the square root of a value gives two answers.
$x=\pm\frac{\sqrt{17}}{4}+\frac{7}{4}$	**8** Add $\frac{7}{4}$ to both sides.
So $x=\frac{7}{4}-\frac{\sqrt{17}}{4}$ or $x=\frac{7}{4}+\frac{\sqrt{17}}{4}$	**9** Write down both the solutions.

Practice

1 Solve by completing the square.

a $x^2-4x-3=0$ **b** $x^2-10x+4=0$

c $x^2+8x-5=0$ **d** $x^2-2x-6=0$

e $2x^2+8x-5=0$ **f** $5x^2+3x-4=0$

2 Solve by completing the square.

a $(x-4)(x+2)=5$

b $2x^2+6x-7=0$

c $x^2-5x+3=0$

> **Hint**
>
> Get all terms onto one side of the equation.

Solving quadratic equations by using the formula

A LEVEL LINKS
Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

- Any quadratic equation of the form $ax^2 + bx + c = 0$ can be solved using the formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- If $b^2 - 4ac$ is negative then the quadratic equation does not have any real solutions.
- It is useful to write down the formula before substituting the values for a, b and c.

Examples

Example 7 Solve $x^2 + 6x + 4 = 0$. Give your solutions in surd form.

$a = 1$, $b = 6$, $c = 4$ $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	**1** Identify a, b and c and write down the formula. Remember that $-b \pm \sqrt{b^2 - 4ac}$ is all over $2a$, not just part of it.
$x = \dfrac{-6 \pm \sqrt{6^2 - 4(1)(4)}}{2(1)}$	**2** Substitute $a = 1$, $b = 6$, $c = 4$ into the formula.
$x = \dfrac{-6 \pm \sqrt{20}}{2}$	**3** Simplify. The denominator is 2, but this is only because $a = 1$. The denominator will not always be 2.
$x = \dfrac{-6 \pm 2\sqrt{5}}{2}$	**4** Simplify $\sqrt{20}$. $\sqrt{20} = \sqrt{4 \times 5} = \sqrt{4} \times \sqrt{5} = 2\sqrt{5}$
$x = -3 \pm \sqrt{5}$	**5** Simplify by dividing numerator and denominator by 2.
So $x = -3 - \sqrt{5}$ or $x = \sqrt{5} - 3$	**6** Write down both the solutions.

Example 8 Solve $3x^2 - 7x - 2 = 0$. Give your solutions in surd form.

$a = 3, b = -7, c = -2$ $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	**1** Identify a, b and c, making sure you get the signs right and write down the formula. Remember that $-b \pm \sqrt{b^2 - 4ac}$ is all over $2a$, not just part of it.
$x = \dfrac{-(-7) \pm \sqrt{(-7)^2 - 4(3)(-2)}}{2(3)}$	**2** Substitute $a = 3$, $b = -7$, $c = -2$ into the formula.
$x = \dfrac{7 \pm \sqrt{73}}{6}$	**3** Simplify. The denominator is 6 when $a = 3$. A common mistake is to always write a denominator of 2.
So $x = \dfrac{7 - \sqrt{73}}{6}$ or $x = \dfrac{7 + \sqrt{73}}{6}$	**4** Write down both the solutions.

Practice

1 Solve, giving your solutions in surd form.

a $3x^2 + 6x + 2 = 0$ **b** $2x^2 - 4x - 7 = 0$

2 Solve the equation $x^2 - 7x + 2 = 0$

Give your solutions in the form $\dfrac{a \pm \sqrt{b}}{c}$, where a, b and c are integers.

3 Solve $10x^2 + 3x + 3 = 5$

Give your solution in surd form.

> **Hint**
>
> Get all terms onto one side of the equation.

Extend

4 Choose an appropriate method to solve each quadratic equation, giving your answer in surd form when necessary.

a $4x(x - 1) = 3x - 2$

b $10 = (x + 1)^2$

c $x(3x - 1) = 10$

Sketching quadratic graphs

A LEVEL LINKS
Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

- The graph of the quadratic function $y = ax^2 + bx + c$, where $a \neq 0$, is a curve called a parabola.
- Parabolas have a line of symmetry and a shape as shown.

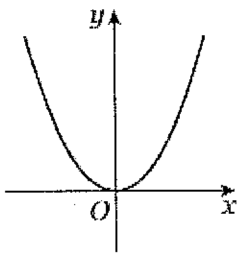
for $a > 0$

for $a < 0$

- To sketch the graph of a function, find the points where the graph intersects the axes.
- To find where the curve intersects the y-axis substitute $x = 0$ into the function.
- To find where the curve intersects the x-axis substitute $y = 0$ into the function.
- At the turning points of a graph the gradient of the curve is 0 and any tangents to the curve at these points are horizontal.
- To find the coordinates of the maximum or minimum point (turning points) of a quadratic curve (parabola) you can use the completed square form of the function.

Examples

Example 1 Sketch the graph of $y = x^2$.

 	The graph of $y = x^2$ is a parabola. When $x = 0$, $y = 0$. $a = 1$ which is greater than zero, so the graph has the shape: 

Example 2 Sketch the graph of $y = x^2 - x - 6$.

When $x = 0$, $y = 0^2 - 0 - 6 = -6$ So the graph intersects the y-axis at $(0, -6)$ When $y = 0$, $x^2 - x - 6 = 0$	**1** Find where the graph intersects the y-axis by substituting $x = 0$. **2** Find where the graph intersects the x-axis by substituting $y = 0$.
$(x + 2)(x - 3) = 0$	**3** Solve the equation by factorising.
$x = -2$ or $x = 3$	**4** Solve $(x + 2) = 0$ and $(x - 3) = 0$.
So, the graph intersects the x-axis at $(-2, 0)$ and $(3, 0)$	**5** $a = 1$ which is greater than zero, so the graph has the shape:  *(continued on next page)*
$x^2 - x - 6 = \left(x - \frac{1}{2}\right)^2 - \frac{1}{4} - 6$	**6** To find the turning point, complete the square.

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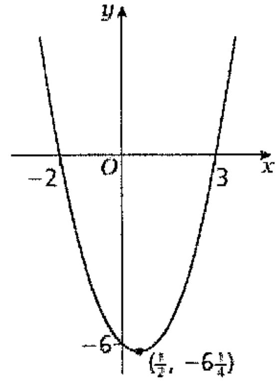
$= \left(x-\frac{1}{2}\right)^2 - \frac{25}{4}$

When $\left(x-\frac{1}{2}\right)^2 = 0$, $x = \frac{1}{2}$ and $y = -\frac{25}{4}$, so the turning point is at the point $\left(\frac{1}{2}, -\frac{25}{4}\right)$



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7 The turning point is the minimum value for this expression and occurs when the term in the bracket is equal to zero.

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Practice

1 Sketch the graph of $y = -x^2$.

2 Sketch each graph, labelling where the curve crosses the axes.

a $y = (x + 2)(x - 1)$ **b** $y = x(x - 3)$ **c** $y = (x + 1)(x + 5)$

3 Sketch each graph, labelling where the curve crosses the axes.

a $y = x^2 - x - 6$ **b** $y = x^2 - 5x + 4$ **c** $y = x^2 - 4$

d $y = x^2 + 4x$ **e** $y = 9 - x^2$ **f** $y = x^2 + 2x - 3$

4 Sketch the graph of $y = 2x^2 + 5x - 3$, labelling where the curve crosses the axes.

Extend

5 Sketch each graph. Label where the curve crosses the axes and write down the coordinates of the turning point.

a $y = x^2 - 5x + 6$ **b** $y = -x^2 + 7x - 12$ **c** $y = -x^2 + 4x$

6 Sketch the graph of $y = x^2 + 2x + 1$. Label where the curve crosses the axes and write down the equation of the line of symmetry.

Solving linear simultaneous equations using the elimination method

A LEVEL LINKS
Scheme of work: 1c. Equations – quadratic/linear simultaneous

Key points

- Two equations are simultaneous when they are both true at the same time.
- Solving simultaneous linear equations in two unknowns involves finding the value of each unknown which works for both equations.
- Make sure that the coefficient of one of the unknowns is the same in both equations.
- Eliminate this equal unknown by either subtracting or adding the two equations.

Examples

Example 1 Solve the simultaneous equations $3x + y = 5$ and $x + y = 1$

$\begin{array}{rl} & 3x + y = 5 \\ - & \;\; x + y = 1 \\ \hline & 2x \qquad = 4 \end{array}$ So $x = 2$	**1** Subtract the second equation from the first equation to eliminate the y term.
Using $x + y = 1$ $2 + y = 1$ So $y = -1$	**2** To find the value of y, substitute $x = 2$ into one of the original equations.
Check: equation 1: $3 \times 2 + (-1) = 5$ YES equation 2: $2 + (-1) = 1$ YES	**3** Substitute the values of x and y into both equations to check your answers.

Example 2 Solve $x + 2y = 13$ and $5x - 2y = 5$ simultaneously.

$\begin{array}{rl} & x + 2y = 13 \\ + & 5x - 2y = \;\; 5 \\ \hline & 6x \qquad = 18 \end{array}$ So $x = 3$	**1** Add the two equations together to eliminate the y term.
Using $x + 2y = 13$ $3 + 2y = 13$ So $y = 5$	**2** To find the value of y, substitute $x = 3$ into one of the original equations.
Check: equation 1: $3 + 2 \times 5 = 13$ YES equation 2: $5 \times 3 - 2 \times 5 = 5$ YES	**3** Substitute the values of x and y into both equations to check your answers.

Example 3 Solve $2x + 3y = 2$ and $5x + 4y = 12$ simultaneously.

$(2x + 3y = 2) \times 4 \rightarrow \quad 8x + 12y = 8$ $(5x + 4y = 12) \times 3 \rightarrow \quad \underline{15x + 12y = 36}$ $7x = 28$ So $x = 4$	**1** Multiply the first equation by 4 and the second equation by 3 to make the coefficient of y the same for both equations. Then subtract the first equation from the second equation to eliminate the y term.
Using $2x + 3y = 2$ $2 \times 4 + 3y = 2$ So $y = -2$	**2** To find the value of y, substitute $x = 4$ into one of the original equations.
Check: equation 1: $2 \times 4 + 3 \times (-2) = 2$ YES equation 2: $5 \times 4 + 4 \times (-2) = 12$ YES	**3** Substitute the values of x and y into both equations to check your answers.

Practice

Solve these simultaneously equations.

1. $4x + y = 8$
   $x + y = 5$

2. $3x + y = 7$
   $3x + 2y = 5$

3. $4x + y = 3$
   $3x - y = 11$

4. $3x + 4y = 7$
   $x - 4y = 5$

5. $2x + y = 11$
   $x - 3y = 9$

6. $2x + 3y = 11$
   $3x + 2y = 4$

Solving linear simultaneous equations using the substitution method

A LEVEL LINKS
Scheme of work: 1c. Equations – quadratic/linear simultaneous
Textbook: Pure Year 1, 3.1 Linear simultaneous equations

Key points

- The subsitution method is the method most commonly used for A level. This is because it is the method used to solve linear and quadratic simultaneous equations.

Examples

Example 4 Solve the simultaneous equations $y = 2x + 1$ and $5x + 3y = 14$

$5x + 3(2x + 1) = 14$	**1** Substitute $2x + 1$ for y into the second equation.
$5x + 6x + 3 = 14$ $11x + 3 = 14$	**2** Expand the brackets and simplify.
$11x = 11$ So $x = 1$	**3** Work out the value of x.
Using $y = 2x + 1$ $y = 2 \times 1 + 1$ So $y = 3$	**4** To find the value of y, substitute $x = 1$ into one of the original equations.
Check: equation 1: $3 = 2 \times 1 + 1$ YES equation 2: $5 \times 1 + 3 \times 3 = 14$ YES	**5** Substitute the values of x and y into both equations to check your answers.

Example 5 Solve $2x - y = 16$ and $4x + 3y = -3$ simultaneously.

$y = 2x - 16$	**1** Rearrange the first equation.
$4x + 3(2x - 16) = -3$	**2** Substitute $2x - 16$ for y into the second equation.
$4x + 6x - 48 = -3$ $10x - 48 = -3$	**3** Expand the brackets and simplify.
$10x = 45$ So $x = 4\frac{1}{2}$	**4** Work out the value of x.
Using $y = 2x - 16$ $y = 2 \times 4\frac{1}{2} - 16$ So $y = -7$	**5** To find the value of y, substitute $x = 4\frac{1}{2}$ into one of the original equations.
Check: equation 1: $2 \times 4\frac{1}{2} - (-7) = 16$ YES equation 2: $4 \times 4\frac{1}{2} + 3 \times (-7) = -3$ YES	**6** Substitute the values of x and y into both equations to check your answers.

Practice

Solve these simultaneous equations.

1 $y = x - 4$
$2x + 5y = 43$

2 $y = 2x - 3$
$5x - 3y = 11$

3 $2y = 4x + 5$
$9x + 5y = 22$

4 $2x = y - 2$
$8x - 5y = -11$

5 $3x + 4y = 8$
$2x - y = -13$

6 $3y = 4x - 7$
$2y = 3x - 4$

7 $3x = y - 1$
$2y - 2x = 3$

8 $3x + 2y + 1 = 0$
$4y = 8 - x$

Extend

9 Solve the simultaneous equations $3x + 5y - 20 = 0$ and $2(x + y) = \dfrac{3(y - x)}{4}$.

Solving linear and quadratic simultaneous equations

A LEVEL LINKS
Scheme of work: 1c. Equations – quadratic/linear simultaneous

Key points

- Make one of the unknowns the subject of the linear equation (rearranging where necessary).
- Use the linear equation to substitute into the quadratic equation.
- There are usually two pairs of solutions.

Examples

Example 1 Solve the simultaneous equations $y = x + 1$ and $x^2 + y^2 = 13$

$x^2 + (x + 1)^2 = 13$	**1** Substitute $x + 1$ for y into the second equation.
$x^2 + x^2 + x + x + 1 = 13$ $2x^2 + 2x + 1 = 13$	**2** Expand the brackets and simplify.
$2x^2 + 2x - 12 = 0$ $(2x - 4)(x + 3) = 0$	**3** Factorise the quadratic equation.
So $x = 2$ or $x = -3$	**4** Work out the values of x.
Using $y = x + 1$ When $x = 2$, $y = 2 + 1 = 3$ When $x = -3$, $y = -3 + 1 = -2$	**5** To find the value of y, substitute both values of x into one of the original equations.
So the solutions are $x = 2,\ y = 3$ and $x = -3, y = -2$	
Check: equation 1: $3 = 2 + 1$ YES and $-2 = -3 + 1$ YES equation 2: $2^2 + 3^2 = 13$ YES and $(-3)^2 + (-2)^2 = 13$ YES	**6** Substitute both pairs of values of x and y into both equations to check your answers.

Example 2 Solve $2x + 3y = 5$ and $2y^2 + xy = 12$ simultaneously.

$x = \dfrac{5-3y}{2}$	**1** Rearrange the first equation.
$2y^2 + \left(\dfrac{5-3y}{2}\right)y = 12$	**2** Substitute $\dfrac{5-3y}{2}$ for x into the second equation. Notice how it is easier to substitute for x than for y.
$2y^2 + \dfrac{5y-3y^2}{2} = 12$ $4y^2 + 5y - 3y^2 = 24$	**3** Expand the brackets and simplify.
$y^2 + 5y - 24 = 0$ $(y + 8)(y - 3) = 0$	**4** Factorise the quadratic equation.
So $y = -8$ or $y = 3$	**5** Work out the values of y.
Using $2x + 3y = 5$ When $y = -8$, $2x + 3 \times (-8) = 5$, $x = 14.5$ When $y = 3$, $2x + 3 \times 3 = 5$, $x = -2$ So the solutions are $x = 14.5$, $y = -8$ and $x = -2$, $y = 3$	**6** To find the value of x, substitute both values of y into one of the original equations.
Check: equation 1: $2 \times 14.5 + 3 \times (-8) = 5$ YES and $2 \times (-2) + 3 \times 3 = 5$ YES equation 2: $2\times(-8)^2 + 14.5\times(-8) = 12$ YES and $2 \times (3)^2 + (-2) \times 3 = 12$ YES	**7** Substitute both pairs of values of x and y into both equations to check your answers.

Practice

Solve these simultaneous equations.

1 $y = 2x + 1$
$x^2 + y^2 = 10$

2 $y = 6 - x$
$x^2 + y^2 = 20$

3 $y = x - 3$
$x^2 + y^2 = 5$

4 $y = 9 - 2x$
$x^2 + y^2 = 17$

5 $y = 3x - 5$
$y = x^2 - 2x + 1$

6 $y = x - 5$
$y = x^2 - 5x - 12$

7 $y = x + 5$
$x^2 + y^2 = 25$

8 $y = 2x - 1$
$x^2 + xy = 24$

9 $y = 2x$
$y^2 - xy = 8$

10 $2x + y = 11$
$xy = 15$

Extend

11 $x - y = 1$
$x^2 + y^2 = 3$

12 $y - x = 2$
$x^2 + xy = 3$

Solving simultaneous equations graphically

A LEVEL LINKS
Scheme of work: 1c. Equations – quadratic/linear simultaneous

Key points

- You can solve any pair of simultaneous equations by drawing the graph of both equations and finding the point/points of intersection.

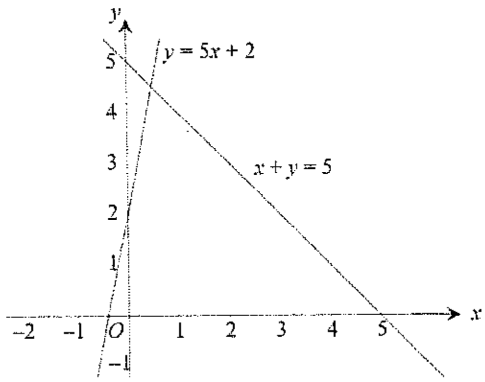
Examples

Example 1 Solve the simultaneous equations $y = 5x + 2$ and $x + y = 5$ graphically.

$y = 5 - x$

1 Rearrange the equation $x + y = 5$ to make y the subject.

$y = 5 - x$ has gradient -1 and y-intercept 5.
$y = 5x + 2$ has gradient 5 and y-intercept 2.



2 Plot both graphs on the same grid using the gradients and y-intercepts.

Lines intersect at
$x = 0.5, y = 4.5$

3 The solutions of the simultaneous equations are the point of intersection.

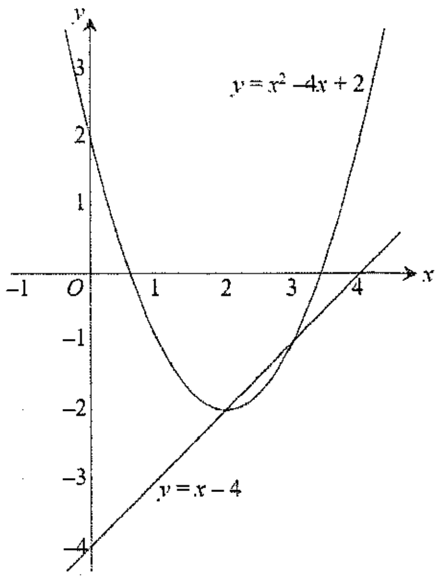
Check:
First equation $y = 5x + 2$:
$4.5 = 5 \times 0.5 + 2$ YES
Second equation $x + y = 5$:
$0.5 + 4.5 = 5$ YES

4 Check your solutions by substituting the values into both equations.

Example 2 Solve the simultaneous equations $y = x - 4$ and $y = x^2 - 4x + 2$ graphically.

x	0	1	2	3	4
y	2	–1	–2	–1	2

1 Construct a table of values and calculate the points for the quadratic equation.



2 Plot the graph.

3 Plot the linear graph on the same grid using the gradient and y-intercept.
$y = x - 4$ has gradient 1 and y-intercept –4.

The line and curve intersect at
$x = 3, y = -1$ and $x = 2, y = -2$

4 The solutions of the simultaneous equations are the points of intersection.

Check:
First equation $y = x - 4$:
$-1 = 3 - 4$ YES
$-2 = 2 - 4$ YES
Second equation $y = x^2 - 4x + 2$:
$-1 = 3^2 - 4 \times 3 + 2$ YES
$-2 = 2^2 - 4 \times 2 + 2$ YES

5 Check your solutions by substituting the values into both equations.

Practice

1 Solve these pairs of simultaneous equations graphically.

a $y = 3x - 1$ and $y = x + 3$

b $y = x - 5$ and $y = 7 - 5x$

c $y = 3x + 4$ and $y = 2 - x$

2 Solve these pairs of simultaneous equations graphically.

a $x + y = 0$ and $y = 2x + 6$

b $4x + 2y = 3$ and $y = 3x - 1$

c $2x + y + 4 = 0$ and $2y = 3x - 1$

Hint

Rearrange the equation to make y the subject.

3 Solve these pairs of simultaneous equations graphically.

a $y = x - 1$ and $y = x^2 - 4x + 3$

b $y = 1 - 3x$ and $y = x^2 - 3x - 3$

c $y = 3 - x$ and $y = x^2 + 2x + 5$

4 Solve the simultaneous equations $x + y = 1$ and $x^2 + y^2 = 25$ graphically.

Extend

5 **a** Solve the simultaneous equations $2x + y = 3$ and $x^2 + y = 4$

i graphically

ii algebraically to 2 decimal places.

b Which method gives the more accurate solutions? Explain your answer.

Linear inequalities

A LEVEL LINKS
Scheme of work: 1d. Inequalities – linear and quadratic (including graphical solutions)

Key points

- Solving linear inequalities uses similar methods to those for solving linear equations.
- When you multiply or divide an inequality by a negative number you need to reverse the inequality sign, e.g. $<$ becomes $>$.

Examples

Example 1 Solve $-8 \leq 4x < 16$

$-8 \leq 4x < 16$ $-2 \leq x < 4$	Divide all three terms by 4.

Example 2 Solve $4 \leq 5x < 10$

$4 \leq 5x < 10$ $\frac{4}{5} \leq x < 2$	Divide all three terms by 5.

Example 3 Solve $2x - 5 < 7$

$2x - 5 < 7$ $2x < 12$ $x < 6$	**1** Add 5 to both sides. **2** Divide both sides by 2.

Example 4 Solve $2 - 5x \geq -8$

$2 - 5x \geq -8$ $-5x \geq -10$ $x \leq 2$	**1** Subtract 2 from both sides. **2** Divide both sides by -5. Remember to reverse the inequality when dividing by a negative number.

Example 5 Solve $4(x - 2) > 3(9 - x)$

$4(x - 2) > 3(9 - x)$ $4x - 8 > 27 - 3x$ $7x - 8 > 27$ $7x > 35$ $x > 5$	**1** Expand the brackets. **2** Add $3x$ to both sides. **3** Add 8 to both sides. **4** Divide both sides by 7.

Practice

1 Solve these inequalities.

a $4x > 16$ **b** $5x - 7 \leq 3$ **c** $1 \geq 3x + 4$

d $5 - 2x < 12$ **e** $\frac{x}{2} \geq 5$ **f** $8 < 3 - \frac{x}{3}$

2 Solve these inequalities.

a $\frac{x}{5} < -4$ **b** $10 \geq 2x + 3$ **c** $7 - 3x > -5$

3 Solve

a $2 - 4x \geq 18$ **b** $3 \leq 7x + 10 < 45$ **c** $6 - 2x \geq 4$

d $4x + 17 < 2 - x$ **e** $4 - 5x < -3x$ **f** $-4x \geq 24$

4 Solve these inequalities.

a $3t + 1 < t + 6$ **b** $2(3n - 1) \geq n + 5$

5 Solve.

a $3(2 - x) > 2(4 - x) + 4$ **b** $5(4 - x) > 3(5 - x) + 2$

Extend

6 Find the set of values of x for which $2x + 1 > 11$ and $4x - 2 > 16 - 2x$.

Quadratic inequalities

A LEVEL LINKS
Scheme of work: 1d. Inequalities – linear and quadratic (including graphical solutions)

Key points

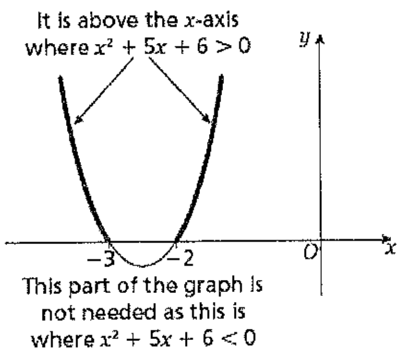
- First replace the inequality sign by = and solve the quadratic equation.
- Sketch the graph of the quadratic function.
- Use the graph to find the values which satisfy the quadratic inequality.

Examples

Example 1 Find the set of values of x which satisfy $x^2 + 5x + 6 > 0$

$x^2 + 5x + 6 = 0$
$(x + 3)(x + 2) = 0$
$x = -3$ or $x = -2$

1 Solve the quadratic equation by factorising.



2 Sketch the graph of $y = (x + 3)(x + 2)$

3 Identify on the graph where $x^2 + 5x + 6 > 0$, i.e. where $y > 0$

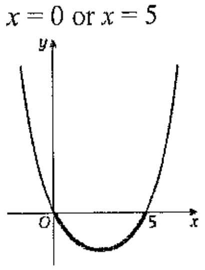
$x < -3$ or $x > -2$

4 Write down the values which satisfy the inequality $x^2 + 5x + 6 > 0$

Example 2 Find the set of values of x which satisfy $x^2 - 5x \leq 0$

$x^2 - 5x = 0$
$x(x - 5) = 0$
$x = 0$ or $x = 5$

1 Solve the quadratic equation by factorising.



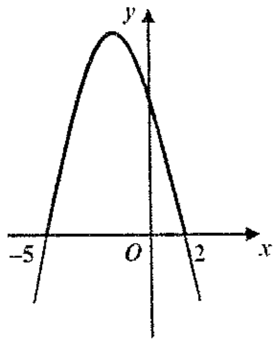
2 Sketch the graph of $y = x(x - 5)$

3 Identify on the graph where $x^2 - 5x \leq 0$, i.e. where $y \leq 0$

$0 \leq x \leq 5$

4 Write down the values which satisfy the inequality $x^2 - 5x \leq 0$

Example 3 Find the set of values of x which satisfy $-x^2 - 3x + 10 \geq 0$

$-x^2 - 3x + 10 = 0$ $(-x + 2)(x + 5) = 0$ $x = 2$ or $x = -5$	**1** Solve the quadratic equation by factorising.
	**2** Sketch the graph of $y = (-x + 2)(x + 5) = 0$ **3** Identify on the graph where $-x^2 - 3x + 10 \geq 0$, i.e. where $y \geq 0$
$-5 \leq x \leq 2$	**3** Write down the values which satisfy the inequality $-x^2 - 3x + 10 \geq 0$

Practice

1 Find the set of values of x for which $(x + 7)(x - 4) \leq 0$

2 Find the set of values of x for which $x^2 - 4x - 12 \geq 0$

3 Find the set of values of x for which $2x^2 - 7x + 3 < 0$

4 Find the set of values of x for which $4x^2 + 4x - 3 > 0$

5 Find the set of values of x for which $12 + x - x^2 \geq 0$

Extend

Find the set of values which satisfy the following inequalities.

6 $x^2 + x \leq 6$

7 $x(2x - 9) < -10$

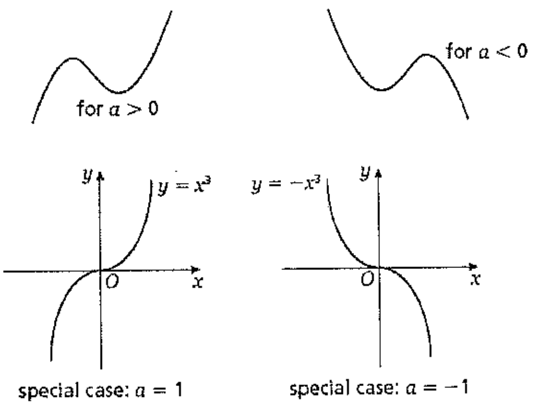
8 $6x^2 \geq 15 + x$

Sketching cubic and reciprocal graphs

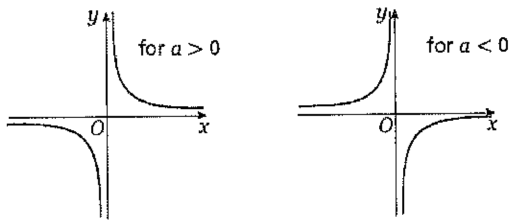
A LEVEL LINKS
Scheme of work: 1e. Graphs – cubic, quartic and reciprocal

Key points

- The graph of a cubic function, which can be written in the form $y = ax^3 + bx^2 + cx + d$, where $a \neq 0$, has one of the shapes shown here.



- The graph of a reciprocal function of the form $y = \frac{a}{x}$ has one of the shapes shown here.

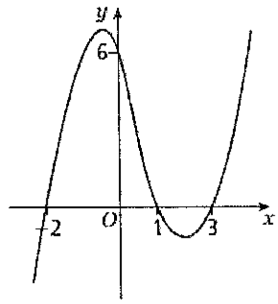


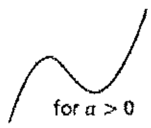
- To sketch the graph of a function, find the points where the graph intersects the axes.
- To find where the curve intersects the y-axis substitute $x = 0$ into the function.
- To find where the curve intersects the x-axis substitute $y = 0$ into the function.
- Where appropriate, mark and label the asymptotes on the graph.
- Asymptotes are lines (usually horizontal or vertical) which the curve gets closer to but never touches or crosses. Asymptotes usually occur with reciprocal functions. For example, the asymptotes for the graph of $y = \frac{a}{x}$ are the two axes (the lines $y = 0$ and $x = 0$).
- At the turning points of a graph the gradient of the curve is 0 and any tangents to the curve at these points are horizontal.
- A double root is when two of the solutions are equal. For example $(x - 3)^2(x + 2)$ has a double root at $x = 3$.
- When there is a double root, this is one of the turning points of a cubic function.

Examples

Example 1 Sketch the graph of $y = (x - 3)(x - 1)(x + 2)$

<table>
<tr><td colspan="2">To sketch a cubic curve find intersects with both axes and use the key points above for the correct shape.</td></tr>
<tr><td>When $x = 0$, $y = (0 - 3)(0 - 1)(0 + 2)$
$= (-3) \times (-1) \times 2 = 6$
The graph intersects the y-axis at $(0, 6)$

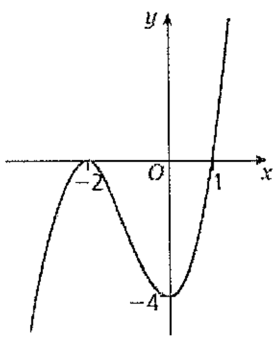
When $y = 0$, $(x - 3)(x - 1)(x + 2) = 0$
So $x = 3$, $x = 1$ or $x = -2$
The graph intersects the x-axis at
$(-2, 0)$, $(1, 0)$ and $(3, 0)$

</td>
<td>1 Find where the graph intersects the axes by substituting $x = 0$ and $y = 0$. Make sure you get the coordinates the right way around, (x, y).
2 Solve the equation by solving $x - 3 = 0$, $x - 1 = 0$ and $x + 2 = 0$

3 Sketch the graph.
$a = 1 > 0$ so the graph has the shape:

</td></tr>
</table>

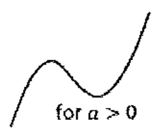
Example 2 Sketch the graph of $y = (x + 2)^2(x - 1)$

<table>
<tr><td colspan="2">To sketch a cubic curve find intersects with both axes and use the key points above for the correct shape.</td></tr>
<tr><td>When $x = 0$, $y = (0 + 2)^2(0 - 1)$
$= 2^2 \times (-1) = -4$
The graph intersects the y-axis at $(0, -4)$

When $y = 0$, $(x + 2)^2(x - 1) = 0$
So $x = -2$ or $x = 1$

$(-2, 0)$ is a turning point as $x = -2$ is a double root.
The graph crosses the x-axis at $(1, 0)$

</td>
<td>1 Find where the graph intersects the axes by substituting $x = 0$ and $y = 0$.

2 Solve the equation by solving $x + 2 = 0$ and $x - 1 = 0$

3 $a = 1 > 0$ so the graph has the shape:

</td></tr>
</table>

Practice

1 Here are six equations.

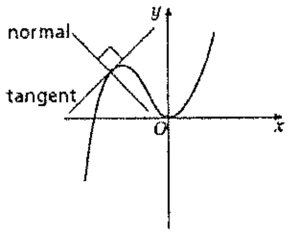
A $y = \dfrac{5}{x}$ **B** $y = x^2 + 3x - 10$ **C** $y = x^3 + 3x^2$

D $y = 1 - 3x^2 - x^3$ **E** $y = x^3 - 3x^2 - 1$ **F** $x + y = 5$

> **Hint**
>
> Find where each of the cubic equations cross the y-axis.

Here are six graphs.

i



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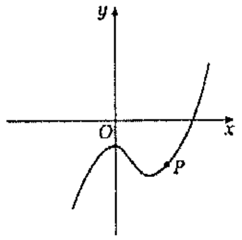
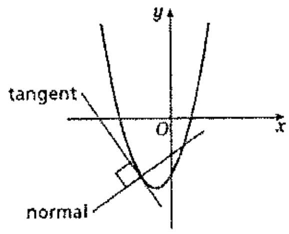


iii



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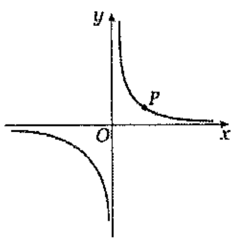

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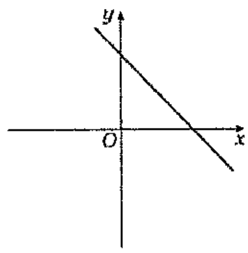
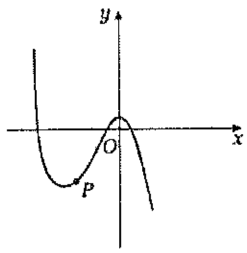


vi



a Match each graph to its equation.

b Copy the graphs ii, iv and vi and draw the tangent and normal each at point P.

Sketch the following graphs

2 $y = 2x^3$

3 $y = x(x - 2)(x + 2)$

4 $y = (x + 1)(x + 4)(x - 3)$

5 $y = (x + 1)(x - 2)(1 - x)$

6 $y = (x - 3)^2(x + 1)$

7 $y = (x - 1)^2(x - 2)$

8 $y = \dfrac{3}{x}$

> **Hint:** Look at the shape of $y = \dfrac{a}{x}$ in the second key point.

9 $y = -\dfrac{2}{x}$

Extend

10 Sketch the graph of $y = \dfrac{1}{x+2}$

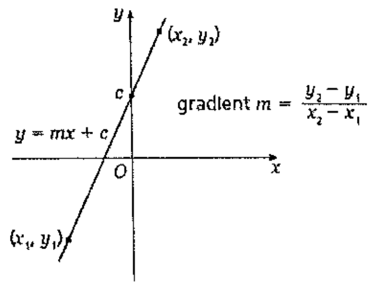
11 Sketch the graph of $y = \dfrac{1}{x-1}$

Straight line graphs

A LEVEL LINKS
Scheme of work: 2a. Straight-line graphs, parallel/perpendicular, length and area problems

Key points

- A straight line has the equation $y = mx + c$, where m is the gradient and c is the y-intercept (where $x = 0$).
- The equation of a straight line can be written in the form $ax + by + c = 0$, where a, b and c are integers.
- When given the coordinates (x_1, y_1) and (x_2, y_2) of two points on a line the gradient is calculated using the formula $m = \dfrac{y_2 - y_1}{x_2 - x_1}$



Examples

Example 1 A straight line has gradient $-\frac{1}{2}$ and y-intercept 3.

Write the equation of the line in the form $ax + by + c = 0$.

$m = -\frac{1}{2}$ and $c = 3$ So $y = -\frac{1}{2}x + 3$ $\frac{1}{2}x + y - 3 = 0$ $x + 2y - 6 = 0$	**1** A straight line has equation $y = mx + c$. Substitute the gradient and y-intercept given in the question into this equation. **2** Rearrange the equation so all the terms are on one side and 0 is on the other side. **3** Multiply both sides by 2 to eliminate the denominator.

Example 2 Find the gradient and the y-intercept of the line with the equation $3y - 2x + 4 = 0$.

$3y - 2x + 4 = 0$ $3y = 2x - 4$ $y = \frac{2}{3}x - \frac{4}{3}$ Gradient $= m = \frac{2}{3}$ y-intercept $= c = -\frac{4}{3}$	**1** Make y the subject of the equation. **2** Divide all the terms by three to get the equation in the form $y = \ldots$ **3** In the form $y = mx + c$, the gradient is m and the y-intercept is c.

Example 3 Find the equation of the line which passes through the point (5, 13) and has gradient 3.

$m = 3$ $y = 3x + c$	1 Substitute the gradient given in the question into the equation of a straight line $y = mx + c$.
$13 = 3 \times 5 + c$	2 Substitute the coordinates $x = 5$ and $y = 13$ into the equation.
$13 = 15 + c$ $c = -2$	3 Simplify and solve the equation.
$y = 3x - 2$	4 Substitute $c = -2$ into the equation $y = 3x + c$

Example 4 Find the equation of the line passing through the points with coordinates (2, 4) and (8, 7).

$x_1 = 2,\ x_2 = 8,\ y_1 = 4$ and $y_2 = 7$ $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-4}{8-2} = \frac{3}{6} = \frac{1}{2}$	1 Substitute the coordinates into the equation $m = \frac{y_2 - y_1}{x_2 - x_1}$ to work out the gradient of the line.
$y = \frac{1}{2}x + c$	2 Substitute the gradient into the equation of a straight line $y = mx + c$.
$4 = \frac{1}{2} \times 2 + c$	3 Substitute the coordinates of either point into the equation.
$c = 3$	4 Simplify and solve the equation.
$y = \frac{1}{2}x + 3$	5 Substitute $c = 3$ into the equation $y = \frac{1}{2}x + c$

Practice

1 Find the gradient and the y-intercept of the following equations.

a $y = 3x + 5$ **b** $y = -\frac{1}{2}x - 7$

c $2y = 4x - 3$ **d** $x + y = 5$

e $2x - 3y - 7 = 0$ **f** $5x + y - 4 = 0$

> **Hint**
> Rearrange the equations to the form $y = mx + c$

2 Copy and complete the table, giving the equation of the line in the form $y = mx + c$.

Gradient	y-intercept	Equation of the line
5	0	
–3	2	
4	–7	

3 Find, in the form $ax + by + c = 0$ where a, b and c are integers, an equation for each of the lines with the following gradients and y-intercepts.

a gradient $-\frac{1}{2}$, y-intercept -7 **b** gradient 2, y-intercept 0

c gradient $\frac{2}{3}$, y-intercept 4 **d** gradient -1.2, y-intercept -2

4 Write an equation for the line which passes though the point $(2, 5)$ and has gradient 4.

5 Write an equation for the line which passes through the point $(6, 3)$ and has gradient $-\frac{2}{3}$

6 Write an equation for the line passing through each of the following pairs of points.

a $(4, 5)$, $(10, 17)$ **b** $(0, 6)$, $(-4, 8)$

c $(-1, -7)$, $(5, 23)$ **d** $(3, 10)$, $(4, 7)$

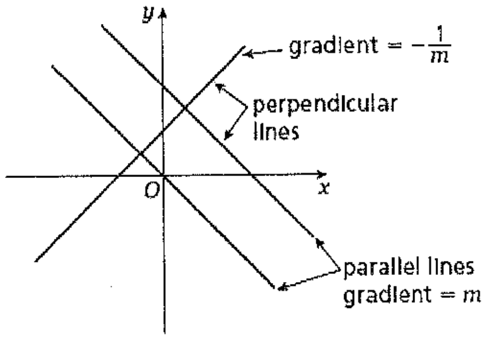
Extend

7 The equation of a line is $2y + 3x - 6 = 0$.
Write as much information as possible about this line.

Parallel and perpendicular lines

A LEVEL LINKS
Scheme of work: 2a. Straight-line graphs, parallel/perpendicular, length and area problems

Key points



- When lines are parallel they have the same gradient.
- A line perpendicular to the line with equation $y = mx + c$ has gradient $-\frac{1}{m}$.

Examples

Example 1 Find the equation of the line parallel to $y = 2x + 4$ which passes through the point (4, 9).

$y = 2x + 4$ $m = 2$ $y = 2x + c$	**1** As the lines are parallel they have the same gradient. **2** Substitute $m = 2$ into the equation of a straight line $y = mx + c$.
$9 = 2 \times 4 + c$	**3** Substitute the coordinates into the equation $y = 2x + c$
$9 = 8 + c$ $c = 1$	**4** Simplify and solve the equation.
$y = 2x + 1$	**5** Substitute $c = 1$ into the equation $y = 2x + c$

Example 2 Find the equation of the line perpendicular to $y = 2x - 3$ which passes through the point (−2, 5).

$y = 2x - 3$ $m = 2$ $-\frac{1}{m} = -\frac{1}{2}$	**1** As the lines are perpendicular, the gradient of the perpendicular line is $-\frac{1}{m}$.
$y = -\frac{1}{2}x + c$	**2** Substitute $m = -\frac{1}{2}$ into $y = mx + c$.
$5 = -\frac{1}{2} \times (-2) + c$	**3** Substitute the coordinates (−2, 5) into the equation $y = -\frac{1}{2}x + c$
$5 = 1 + c$ $c = 4$	**4** Simplify and solve the equation.
$y = -\frac{1}{2}x + 4$	**5** Substitute $c = 4$ into $y = -\frac{1}{2}x + c$.

Example 3 A line passes through the points (0, 5) and (9, −1).
Find the equation of the line which is perpendicular to the line and passes through its midpoint.

$x_1 = 0$, $x_2 = 9$, $y_1 = 5$ and $y_2 = -1$ $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-5}{9-0}$ $= \frac{-6}{9} = -\frac{2}{3}$	**1** Substitute the coordinates into the equation $m = \frac{y_2 - y_1}{x_2 - x_1}$ to work out the gradient of the line.
$-\frac{1}{m} = \frac{3}{2}$	**2** As the lines are perpendicular, the gradient of the perpendicular line is $-\frac{1}{m}$.
$y = \frac{3}{2}x + c$	**3** Substitute the gradient into the equation $y = mx + c$.
Midpoint $= \left(\frac{0+9}{2}, \frac{5+(-1)}{2}\right) = \left(\frac{9}{2}, 2\right)$	**4** Work out the coordinates of the midpoint of the line.
$2 = \frac{3}{2} \times \frac{9}{2} + c$	**5** Substitute the coordinates of the midpoint into the equation.
$c = -\frac{19}{4}$	**6** Simplify and solve the equation.
$y = \frac{3}{2}x - \frac{19}{4}$	**7** Substitute $c = -\frac{19}{4}$ into the equation $y = \frac{3}{2}x + c$.

Practice

1 Find the equation of the line parallel to each of the given lines and which passes through each of the given points.

a $y = 3x + 1$ (3, 2) **b** $y = 3 - 2x$ (1, 3)

c $2x + 4y + 3 = 0$ (6, −3) **d** $2y - 3x + 2 = 0$ (8, 20)

2 Find the equation of the line perpendicular to $y = \frac{1}{2}x - 3$ which passes through the point (−5, 3).

> **Hint**
> If $m = \frac{a}{b}$ then the negative reciprocal $-\frac{1}{m} = -\frac{b}{a}$

3 Find the equation of the line perpendicular to each of the given lines and which passes through each of the given points.

a $y = 2x - 6$ (4, 0) **b** $y = -\frac{1}{3}x + \frac{1}{2}$ (2, 13)

c $x - 4y - 4 = 0$ (5, 15) **d** $5y + 2x - 5 = 0$ (6, 7)

4 In each case find an equation for the line passing through the origin which is also perpendicular to the line joining the two points given.

a (4, 3), (−2, −9) **b** (0, 3), (−10, 8)

Extend

5 Work out whether these pairs of lines are parallel, perpendicular or neither.

a $y = 2x + 3$
$y = 2x - 7$

b $y = 3x$
$2x + y - 3 = 0$

c $y = 4x - 3$
$4y + x = 2$

d $3x - y + 5 = 0$
$x + 3y = 1$

e $2x + 5y - 1 = 0$
$y = 2x + 7$

f $2x - y = 6$
$6x - 3y + 3 = 0$

6 The straight line $\mathbf{L_1}$ passes through the points A and B with coordinates $(-4, 4)$ and $(2, 1)$, respectively.

a Find the equation of $\mathbf{L_1}$ in the form $ax + by + c = 0$

The line $\mathbf{L_2}$ is parallel to the line $\mathbf{L_1}$ and passes through the point C with coordinates $(-8, 3)$.

b Find the equation of $\mathbf{L_2}$ in the form $ax + by + c = 0$

The line $\mathbf{L_3}$ is perpendicular to the line $\mathbf{L_1}$ and passes through the origin.

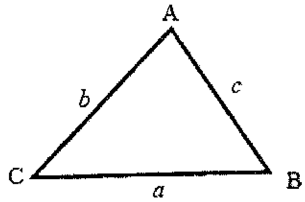
c Find an equation of $\mathbf{L_3}$

The cosine rule

A LEVEL LINKS
Scheme of work: 4a. Trigonometric ratios and graphs
Textbook: Pure Year 1, 9.1 The cosine rule

Key points

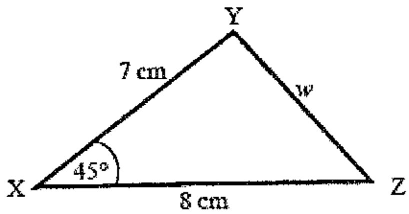
- a is the side opposite angle A.
  b is the side opposite angle B.
  c is the side opposite angle C.

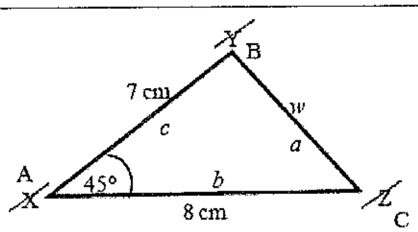


- You can use the cosine rule to find the length of a side when two sides and the included angle are given.
- To calculate an unknown side use the formula $a^2 = b^2 + c^2 - 2bc\cos A$.
- Alternatively, you can use the cosine rule to find an unknown angle if the lengths of all three sides are given.
- To calculate an unknown angle use the formula $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$.

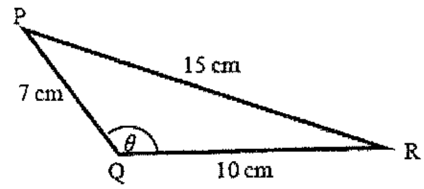
Examples

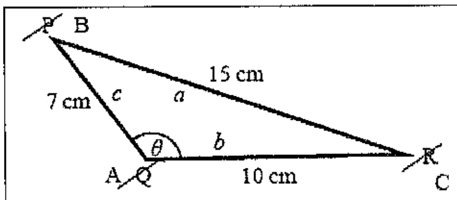
Example 4 Work out the length of side w.
Give your answer correct to 3 significant figures.



	**1** Always start by labelling the angles and sides.
$a^2 = b^2 + c^2 - 2bc\cos A$	**2** Write the cosine rule to find the side.
$w^2 = 8^2 + 7^2 - 2 \times 8 \times 7 \times \cos 45°$	**3** Substitute the values a, b and A into the formula.
$w^2 = 33.804\,040\,51...$ $w = \sqrt{33.80404051}$	**4** Use a calculator to find w^2 and then w.
$w = 5.81$ cm	**5** Round your final answer to 3 significant figures and write the units in your answer.

Example 5 Work out the size of angle θ.
Give your answer correct to 1 decimal place.

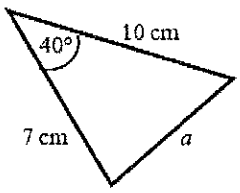


	**1** Always start by labelling the angles and sides.
$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$	**2** Write the cosine rule to find the angle.
$\cos\theta = \frac{10^2 + 7^2 - 15^2}{2 \times 10 \times 7}$	**3** Substitute the values a, b and c into the formula.
$\cos\theta = \frac{-76}{140}$	**4** Use $\cos^{-1}$ to find the angle.
$\theta = 122.878\,349...$	**5** Use your calculator to work out $\cos^{-1}(-76 \div 140)$.
$\theta = 122.9°$	**6** Round your answer to 1 decimal place and write the units in your answer.

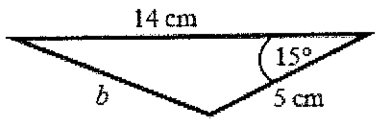
Practice

1 Work out the length of the unknown side in each triangle.
Give your answers correct to 3 significant figures.

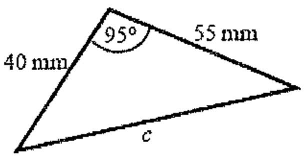
a



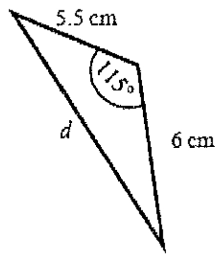
b



c

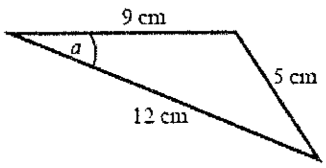


d

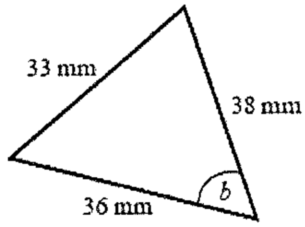


2 Calculate the angles labelled θ in each triangle.
Give your answer correct to 1 decimal place.

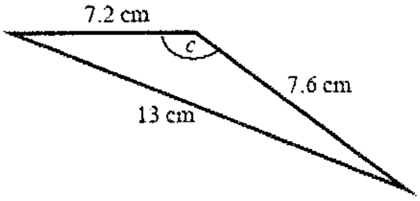
a



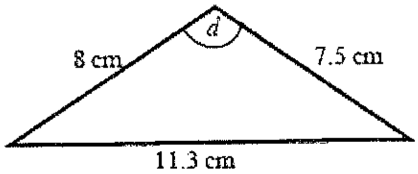
b



c

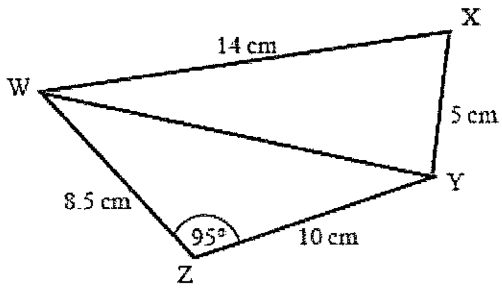


d



3 **a** Work out the length of WY.
Give your answer correct to 3 significant figures.

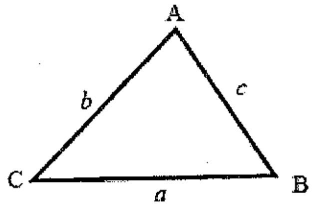
b Work out the size of angle WXY.
Give your answer correct to 1 decimal place.



The sine rule

A LEVEL LINKS
Scheme of work: 4a. Trigonometric ratios and graphs
Textbook: Pure Year 1, 9.2 The sine rule

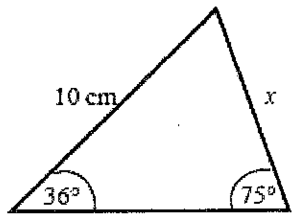
Key points

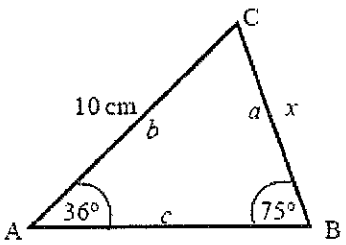


- a is the side opposite angle A.
  b is the side opposite angle B.
  c is the side opposite angle C.
- You can use the sine rule to find the length of a side when its opposite angle and another opposite side and angle are given.
- To calculate an unknown side use the formula $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.
- Alternatively, you can use the sine rule to find an unknown angle if the opposite side and another opposite side and angle are given.
- To calculate an unknown angle use the formula $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$.

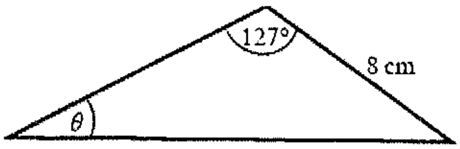
Examples

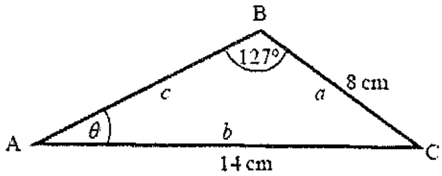
Example 6 Work out the length of side x.
Give your answer correct to 3 significant figures.



	1 Always start by labelling the angles and sides.
$\frac{a}{\sin A} = \frac{b}{\sin B}$	2 Write the sine rule to find the side.
$\frac{x}{\sin 36°} = \frac{10}{\sin 75°}$	3 Substitute the values a, b, A and B into the formula.
$x = \frac{10 \times \sin 36°}{\sin 75°}$	4 Rearrange to make x the subject.
$x = 6.09$ cm	5 Round your answer to 3 significant figures and write the units in your answer.

Example 7 Work out the size of angle θ.
Give your answer correct to 1 decimal place.

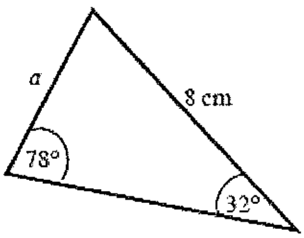


	**1** Always start by labelling the angles and sides.
$\frac{\sin A}{a} = \frac{\sin B}{b}$	**2** Write the sine rule to find the angle.
$\frac{\sin \theta}{8} = \frac{\sin 127°}{14}$	**3** Substitute the values a, b, A and B into the formula.
$\sin \theta = \frac{8 \times \sin 127°}{14}$	**4** Rearrange to make $\sin \theta$ the subject.
$\theta = 27.2°$	**5** Use $\sin^{-1}$ to find the angle. Round your answer to 1 decimal place and write the units in your answer.

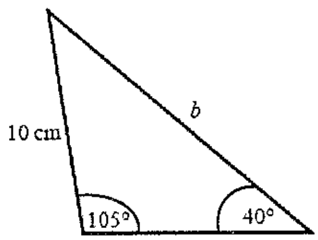
Practice

1 Find the length of the unknown side in each triangle.
Give your answers correct to 3 significant figures.

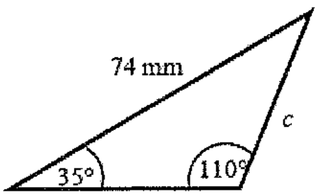
a



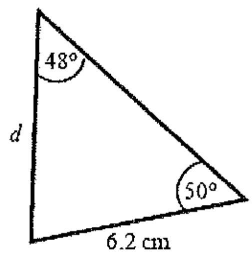
b



c

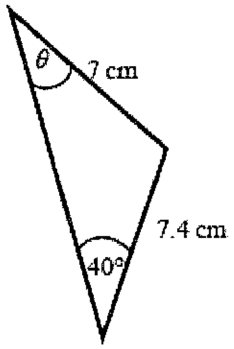


d

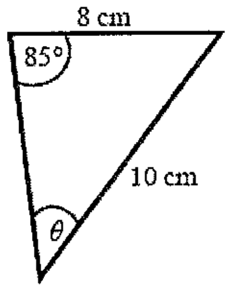


2 Calculate the angles labelled θ in each triangle.
Give your answer correct to 1 decimal place.

a



b



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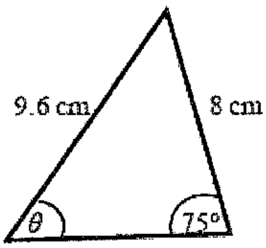

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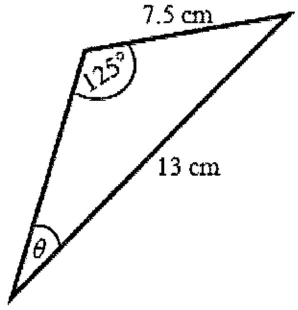
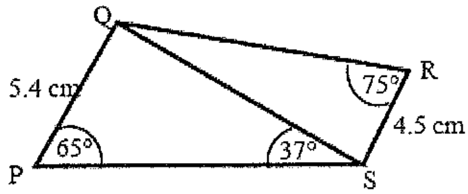


3 **a** Work out the length of QS.
Give your answer correct to 3 significant figures.

b Work out the size of angle RQS.
Give your answer correct to 1 decimal place.

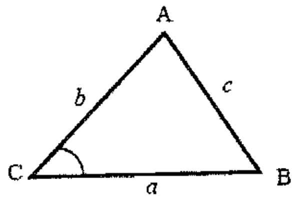


Areas of triangles

A LEVEL LINKS
Scheme of work: 4a. Trigonometric ratios and graphs
Textbook: Pure Year 1, 9.3 Areas of triangles

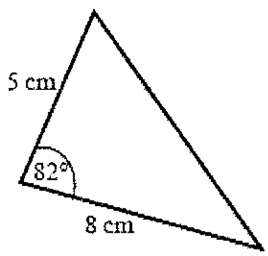
Key points

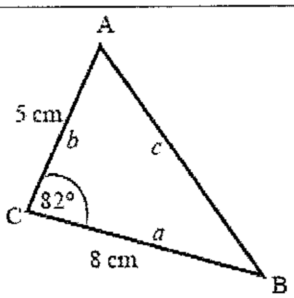
- a is the side opposite angle A.
  b is the side opposite angle B.
  c is the side opposite angle C.
- The area of the triangle is $\frac{1}{2}ab\sin C$.



Examples

Example 8 Find the area of the triangle.



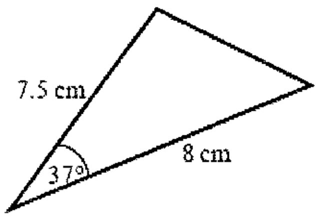


	1 Always start by labelling the sides and angles of the triangle.
Area $= \frac{1}{2}ab\sin C$	**2** State the formula for the area of a triangle.
Area $= \frac{1}{2}\times 8\times 5\times \sin 82°$	**3** Substitute the values of a, b and C into the formula for the area of a triangle.
Area = 19.805 361...	**4** Use a calculator to find the area.
Area = 19.8 cm^2	**5** Round your answer to 3 significant figures and write the units in your answer.

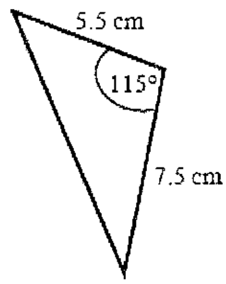
Practice

1 Work out the area of each triangle.
Give your answers correct to 3 significant figures.

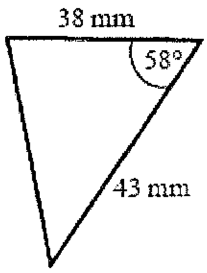
a



b

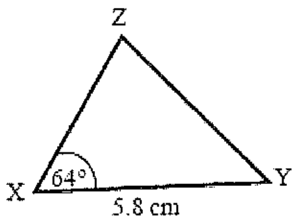


c



2 The area of triangle XYZ is 13.3 cm^2.
Work out the length of XZ.

Hint:
Rearrange the formula to make a side the subject.

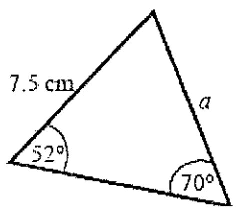


Extend

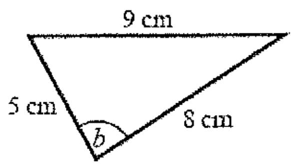
Hint:
For each one, decide whether to use the cosine or sine rule.

3 Find the size of each lettered angle or side.
Give your answers correct to 3 significant figures.

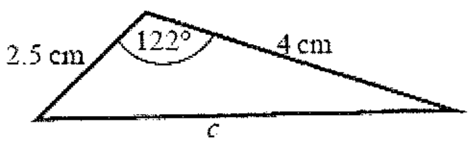
a



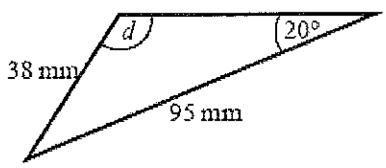
b



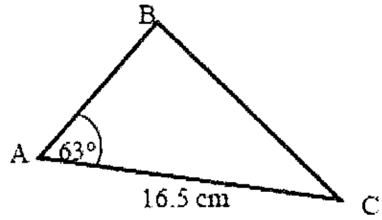
c



d



4 The area of triangle ABC is 86.7 cm^2.
Work out the length of BC.
Give your answer correct to 3 significant figures.



Rearranging equations

A LEVEL LINKS
Scheme of work: 6a. Definition, differentiating polynomials, second derivatives
Textbook: Pure Year 1, 12.1 Gradients of curves

Key points

- To change the subject of a formula, get the terms containing the subject on one side and everything else on the other side.
- You may need to factorise the terms containing the new subject.

Examples

Example 1 Make t the subject of the formula $v = u + at$.

$v = u + at$	**1** Get the terms containing t on one side and everything else on the other side.
$v - u = at$ $t = \dfrac{v-u}{a}$	**2** Divide throughout by a.

Example 2 Make t the subject of the formula $r = 2t - \pi t$.

$r = 2t - \pi t$	**1** All the terms containing t are already on one side and everything else is on the other side.
$r = t(2 - \pi)$	**2** Factorise as t is a common factor.
$t = \dfrac{r}{2-\pi}$	**3** Divide throughout by $2 - \pi$.

Example 3 Make t the subject of the formula $\dfrac{t+r}{5} = \dfrac{3t}{2}$.

$\dfrac{t+r}{5} = \dfrac{3t}{2}$	**1** Remove the fractions first by multiplying throughout by 10.
$2t + 2r = 15t$ $2r = 13t$	**2** Get the terms containing t on one side and everything else on the other side and simplify.
$t = \dfrac{2r}{13}$	**3** Divide throughout by 13.

Example 4 Make t the subject of the formula $r = \frac{3t+5}{t-1}$.

$r = \frac{3t+5}{t-1}$	**1** Remove the fraction first by multiplying throughout by $t - 1$.
$r(t-1) = 3t+5$	**2** Expand the brackets.
$rt - r = 3t + 5$	**3** Get the terms containing t on one side and everything else on the other side.
$rt - 3t = 5 + r$	
$t(r-3) = 5 + r$	**4** Factorise the LHS as t is a common factor.
$t = \frac{5+r}{r-3}$	**5** Divide throughout by $r - 3$.

Practice

Change the subject of each formula to the letter given in the brackets.

1 $C = \pi d$ [d]

2 $P = 2l + 2w$ [w]

3 $D = \frac{S}{T}$ [T]

4 $p = \frac{q-r}{t}$ [t]

5 $u = at - \frac{1}{2}t$ [t]

6 $V = ax + 4x$ [x]

7 $\frac{y-7x}{2} = \frac{7-2y}{3}$ [y]

8 $x = \frac{2a-1}{3-a}$ [a]

9 $x = \frac{b-c}{d}$ [d]

10 $h = \frac{7g-9}{2+g}$ [g]

11 $e(9+x) = 2e + 1$ [e]

12 $y = \frac{2x+3}{4-x}$ [x]

13 Make r the subject of the following formulae.

a $A = \pi r^2$ **b** $V = \frac{4}{3}\pi r^3$ **c** $P = \pi r + 2r$ **d** $V = \frac{2}{3}\pi r^2 h$

14 Make x the subject of the following formulae.

a $\frac{xy}{z} = \frac{ab}{cd}$ **b** $\frac{4\pi cx}{d} = \frac{3z}{py^2}$

15 Make $\sin B$ the subject of the formula $\frac{a}{\sin A} = \frac{b}{\sin B}$

16 Make $\cos B$ the subject of the formula $b^2 = a^2 + c^2 - 2ac\cos B$.

Extend

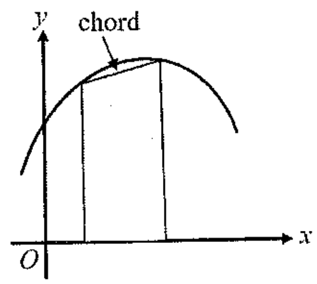
17 Make x the subject of the following equations.

a $\frac{p}{q}(sx+t) = x - 1$ **b** $\frac{p}{q}(ax+2y) = \frac{3p}{q^2}(x-y)$

Area under a graph

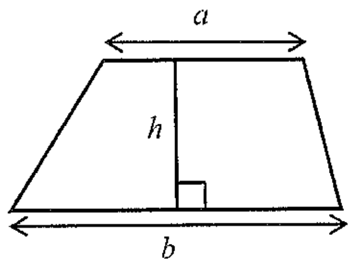
A LEVEL LINKS
Scheme of work: 7b. Definite integrals and areas under curves

Key points

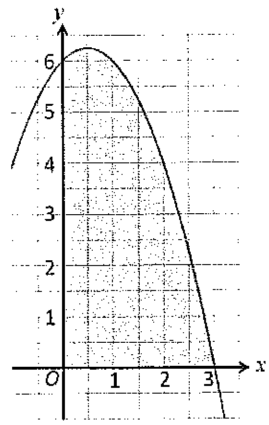


- To estimate the area under a curve, draw a chord between the two points you are finding the area between and straight lines down to the horizontal axis to create a trapezium. The area of the trapezium is an approximation for the area under a curve.

- The area of a trapezium = $\frac{1}{2}h(a+b)$



Examples



Example 1 Estimate the area of the region between the curve $y = (3 - x)(2 + x)$ and the x-axis from $x = 0$ to $x = 3$. Use three strips of width 1 unit.

x	0	1	2	3
$y = (3 - x)(2 + x)$	6	6	4	0

Trapezium 1:
$a_1 = 6 - 0 = 6$, $b_1 = 6 - 0 = 6$
Trapezium 2:
$a_2 = 6 - 0 = 6$, $b_2 = 4 - 0 = 4$
Trapezium 3:
$a_3 = 4 - 0 = 4$, $a_3 = 0 - 0 = 0$

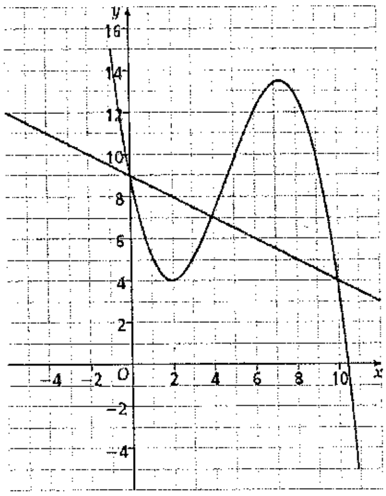
1 Use a table to record the value of y on the curve for each value of x.

2 Work out the dimensions of each trapezium. The distances between the y-values on the curve and the x-axis give the values for a.

(continued on next page)

$\frac{1}{2}h(a_1+b_1)=\frac{1}{2}\times 1(6+6)=6$ $\frac{1}{2}h(a_2+b_2)=\frac{1}{2}\times 1(6+4)=5$ $\frac{1}{2}h(a_3+b_3)=\frac{1}{2}\times 1(4+0)=2$	**3** Work out the area of each trapezium. h = 1 since the width of each trapezium is 1 unit.
Area = 6 + 5 + 2 = 13 units^2	**4** Work out the total area. Remember to give units with your answer.

Example 2 Estimate the shaded area.
Use three strips of width 2 units.



x: 4, 6, 8, 10; y: 7, 12, 13, 4	**1** Use a table to record y on the curve for each value of x.
x: 4, 6, 8, 10; y: 7, 6, 5, 4	**2** Use a table to record y on the straight line for each value of x.
Trapezium 1: $a_1=7-7=0$, $b_1=12-6=6$ Trapezium 2: $a_2=12-6=6$, $b_2=13-5=8$ Trapezium 3: $a_3=13-5=8$, $a_3=4-4=0$	**3** Work out the dimensions of each trapezium. The distances between the y-values on the curve and the y-values on the straight line give the values for a.
$\frac{1}{2}h(a_1+b_1)=\frac{1}{2}\times 2(0+6)=6$ $\frac{1}{2}h(a_2+b_2)=\frac{1}{2}\times 2(6+8)=14$ $\frac{1}{2}h(a_3+b_3)=\frac{1}{2}\times 2(8+0)=8$	**4** Work out the area of each trapezium. h = 2 since the width of each trapezium is 2 units.
Area = 6 + 14 + 8 = 28 units^2	**5** Work out the total area. Remember to give units with your answer.

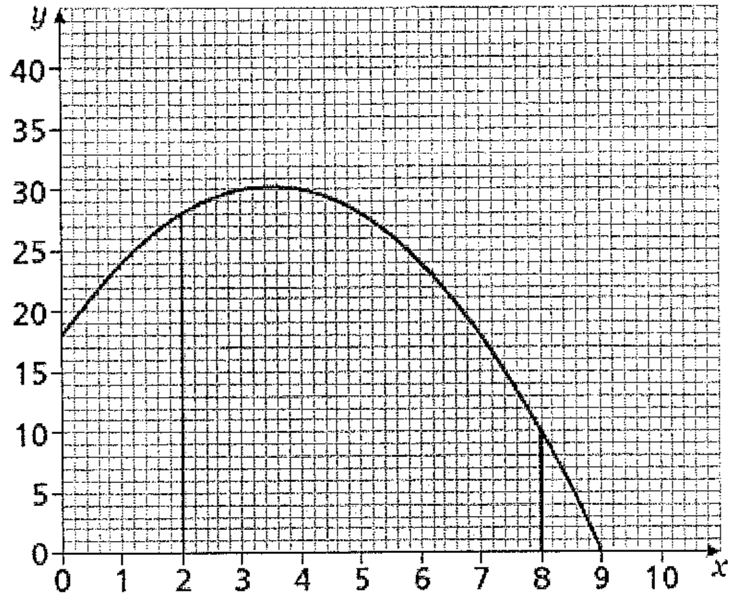
Practice

Hint:

For a full answer, remember to include 'units^2'.

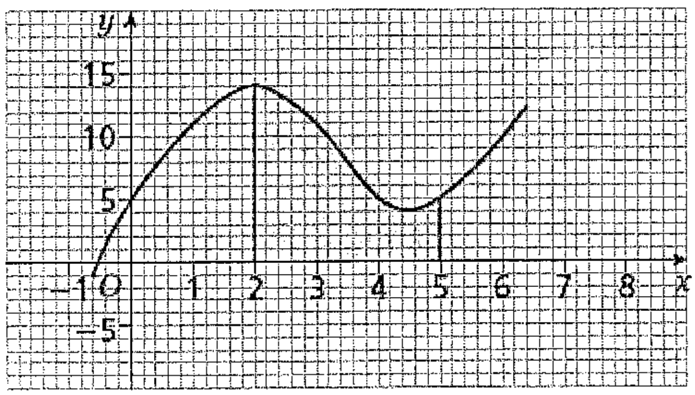
1 Estimate the area of the region between the curve $y = (5 - x)(x + 2)$ and the x-axis from $x = 1$ to $x = 5$.
Use four strips of width 1 unit.

2 Estimate the shaded area shown on the axes.
Use six strips of width 1 unit.



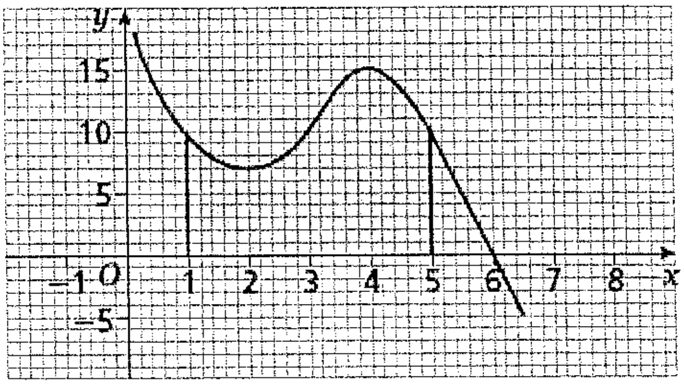
3 Estimate the area of the region between the curve $y = x^2 - 8x + 18$ and the x-axis from $x = 2$ to $x = 6$.
Use four strips of width 1 unit.

4 Estimate the shaded area.
Use six strips of width $\frac{1}{2}$ unit.



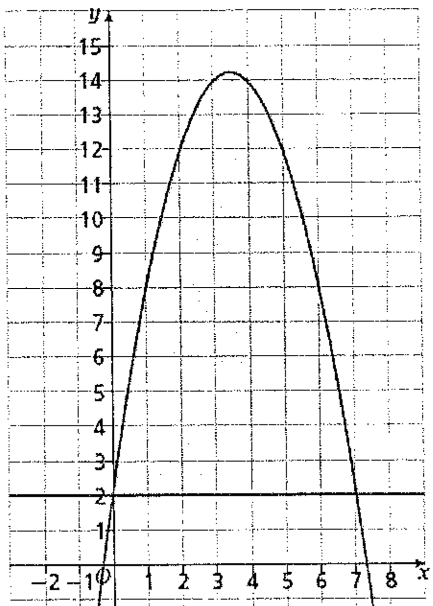
5 Estimate the area of the region between the curve $y = -x^2 - 4x + 5$ and the x-axis from $x = -5$ to $x = 1$.
Use six strips of width 1 unit.

6 Estimate the shaded area.
Use four strips of equal width.



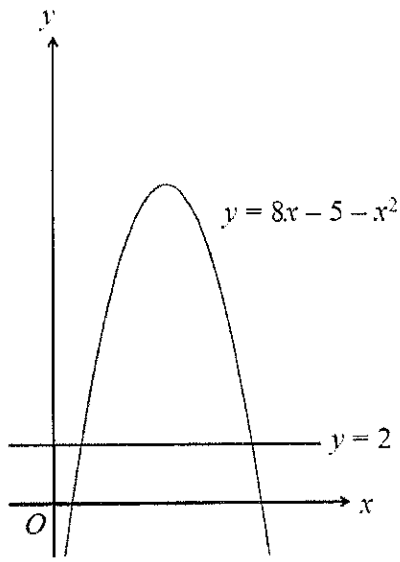
7 Estimate the area of the region between the curve $y = -x^2 + 2x + 15$ and the x-axis from $x = 2$ to $x = 5$.
Use six strips of equal width.

8 Estimate the shaded area.
Use seven strips of equal width.

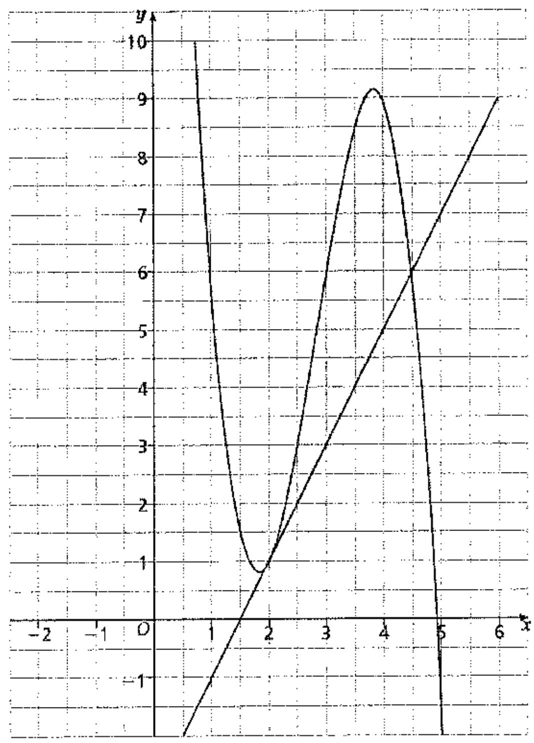


Extend

9 The curve $y = 8x - 5 - x^2$ and the line $y = 2$ are shown in the sketch.
Estimate the shaded area using six strips of equal width.

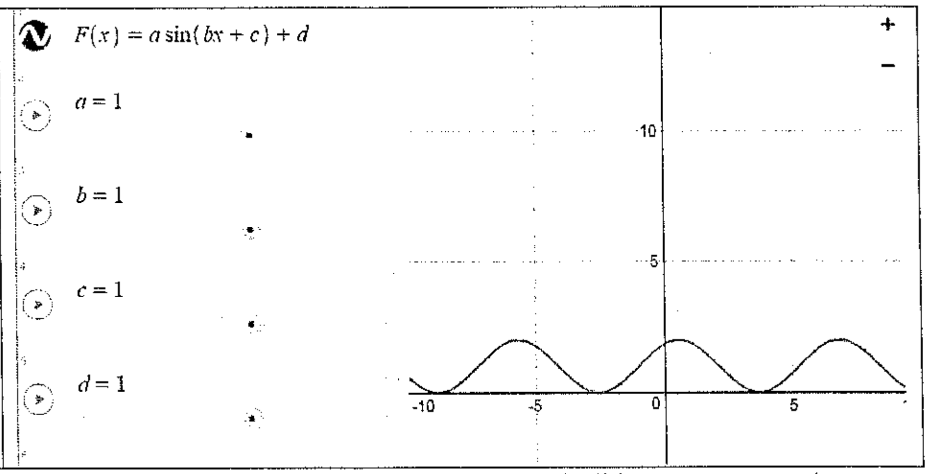


10 Estimate the shaded area using five strips of equal width.



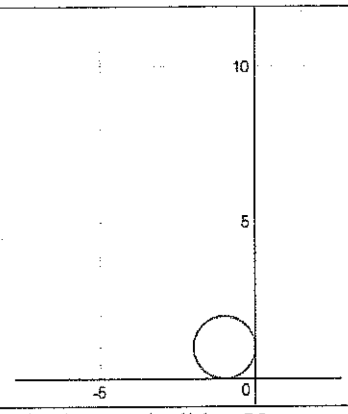
ICT Task: Transforming Graphs

Go to www.desmos.com and select 'Start Graphing'

Type the Function and when it asks you to add sliders, select 'all'. Now use the sliders and explain how the shape of the graph changes for each variable. The screen should look similar to the screen shot here.	

Write descriptions for how the graph changes when you manipulate each slider. You can use the words such as amplitude, wavelength and frequency

	When it increases	When it decreases	When it is zero	When it doubles
Slider a				
Slider b				
Slider c				
Slider d				

Type the Function and when it asks you to add sliders, select 'all'. Now use the sliders and explain how the shape of the graph changes for each variable. The screen should look similar to the screen shot here.	$(x-a)^2+(y-b)^2=r$ $r=1$ $a=-1$ $b=1$ 

Write descriptions for how the graph changes when you manipulate each slider. You can use the words such as amplitude, wavelength and frequency

	When it increases	When it decreases	When it is zero	When it doubles
Slider a				
Slider b				
Slider r				

Challenge – You can make up random functions and see what the graph looks like and how it changes if you add constants and sliders. Here are some examples to explore. Make up your own functions and glue the graphs below.

$p(x-a)^2+q(y-b)^2=r$	$\sin(x+a)+\cos(y+b)=1$	$\sin(x+a)+\cos(y+b)=\frac{1}{xy}$	$\frac{1}{\sin(x+a)}+\frac{1}{\cos(x+b)}=y$

Answers for Part B:

Expanding brackets and simplifying expressions

1 **a** $6x - 3$ **b** $-10pq - 8q^2$
c $-3xy + 2y^2$

2 **a** $21x + 35 + 12x - 48 = 33x - 13$
b $40p - 16 - 12p - 27 = 28p - 43$
c $27s + 9 - 30s + 50 = -3s + 59 = 59 - 3s$
d $8x - 6 - 3x - 5 = 5x - 11$

3 **a** $12x^2 + 24x$ **b** $20k^3 - 48k$
c $10h - 12h^3 - 22h^2$ **d** $21s^2 - 21s^3 - 6s$

4 **a** $-y^2 - 4$ **b** $5x^2 - 11x$
c $2p - 7p^2$ **d** $6b^2$

5 $y - 4$

6 **a** $-1 - 2m$ **b** $5p^3 + 12p^2 + 27p$

7 $7x(3x - 5) = 21x^2 - 35x$

8 **a** $x^2 + 9x + 20$ **b** $x^2 + 10x + 21$
c $x^2 + 5x - 14$ **d** $x^2 - 25$
e $2x^2 + x - 3$ **f** $6x^2 - x - 2$
g $10x^2 - 31x + 15$ **h** $12x^2 + 13x - 14$
i $18x^2 + 39xy + 20y^2$ **j** $x^2 + 10x + 25$
k $4x^2 - 28x + 49$ **l** $16x^2 - 24xy + 9y^2$

9 $2x^2 - 2x + 25$

10 **a** $x^2 - 1 - \dfrac{2}{x^2}$ **b** $x^2 + 2 + \dfrac{1}{x^2}$

11 Expand and simplify.
a $x^3 + 7x^2 + 14x + 8$ **b** $x^3 + 6x^2 - 9x - 14$
c $x^3 - 4x^2 - 59x + 126$ **d** $x^3 - 11x^2 + 38x - 40$
e $2x^3 - 3x^2 - 5x + 6$ **f** $18x^3 - 15x^2 - 4x + 4$
g $x^3 - 9x^2 + 27x - 27$ **h** $x^4 - 12x^3 + 54x^2 - 108x + 81$

Surds and rationalising the denominator

1 a $3\sqrt{5}$ b $5\sqrt{5}$
c $4\sqrt{3}$ d $5\sqrt{7}$
e $10\sqrt{3}$ f $2\sqrt{7}$
g $6\sqrt{2}$ h $9\sqrt{2}$

2 a $15\sqrt{2}$ b $\sqrt{5}$
c $3\sqrt{2}$ d $\sqrt{3}$
e $6\sqrt{7}$ f $5\sqrt{3}$

3 a -1 b $9-\sqrt{3}$
c $10\sqrt{5}-7$ d $26-4\sqrt{2}$

4 a $\frac{\sqrt{5}}{5}$ b $\frac{\sqrt{11}}{11}$
c $\frac{2\sqrt{7}}{7}$ d $\frac{\sqrt{2}}{2}$
e $\sqrt{2}$ f $\sqrt{5}$
g $\frac{\sqrt{3}}{3}$ h $\frac{1}{3}$

5 a $\frac{3+\sqrt{5}}{4}$ b $\frac{2(4-\sqrt{3})}{13}$ c $\frac{6(5+\sqrt{2})}{23}$

6 $x-y$

7 a $3+2\sqrt{2}$ b $\frac{\sqrt{x}+\sqrt{y}}{x-y}$

Rules of indices

1	a	1	b	1	c	1	d	1
2	a	7	b	4	c	5	d	2
3	a	125	b	32	c	343	d	8
4	a	$\frac{1}{25}$	b	$\frac{1}{64}$	c	$\frac{1}{32}$	d	$\frac{1}{36}$
5	a	$\frac{3x^3}{2}$	b	$5x^2$				
	c	$3x$	d	$\frac{y}{2x^2}$				
	e	$y^{\frac{1}{2}}$	f	c^{-3}				
	g	$2x^6$	h	x				
6	a	$\frac{1}{2}$	b	$\frac{1}{9}$	c	$\frac{8}{3}$		
	d	$\frac{1}{4}$	e	$\frac{4}{3}$	f	$\frac{16}{9}$		
7	a	x^{-1}	b	x^{-7}	c	$x^{\frac{1}{4}}$		
	d	$x^{\frac{2}{5}}$	e	$x^{-\frac{1}{3}}$	f	$x^{-\frac{2}{3}}$		
8	a	$\frac{1}{x^3}$	b	1	c	$\sqrt[5]{x}$		
	d	$\sqrt[5]{x^2}$	e	$\frac{1}{\sqrt{x}}$	f	$\frac{1}{\sqrt[4]{x^3}}$		
9	a	$5x^{\frac{1}{2}}$	b	$2x^{-3}$	c	$\frac{1}{3}x^{-4}$		
	d	$2x^{-\frac{1}{2}}$	e	$4x^{-\frac{1}{3}}$	f	$3x^0$		
10	a	$x^3 + x^{-2}$	b	$x^3 + x$	c	$x^{-2} + x^{-7}$		

Factorising expressions

1 a $2x^3y^3(3x - 5y)$ b $7a^3b^2(3b^3 + 5a^2)$
c $5x^2y^2(5 - 2x + 3y)$

2 a $(x + 3)(x + 4)$ b $(x + 7)(x - 2)$
c $(x - 5)(x - 6)$ d $(x - 8)(x + 3)$
e $(x - 9)(x + 2)$ f $(x + 5)(x - 4)$
g $(x - 8)(x + 5)$ h $(x + 7)(x - 4)$

3 a $(6x - 7y)(6x + 7y)$ b $(2x - 9y)(2x + 9y)$
c $2(3a - 10bc)(3a + 10bc)$

4 a $(x - 1)(2x + 3)$ b $(3x + 1)(2x + 5)$
c $(2x + 1)(x + 3)$ d $(3x - 1)(3x - 4)$
e $(5x + 3)(2x + 3)$ f $2(3x - 2)(2x - 5)$

5 a $\frac{2(x+2)}{x-1}$ b $\frac{x}{x-1}$
c $\frac{x+2}{x}$ d $\frac{x}{x+5}$
e $\frac{x+3}{x}$ f $\frac{x}{x-5}$

6 a $\frac{3x+4}{x+7}$ b $\frac{2x+3}{3x-2}$
c $\frac{2-5x}{2x-3}$ d $\frac{3x+1}{x+4}$

7 $(x + 5)$

8 $\frac{4(x+2)}{x-2}$

Completing the square

1 a $(x + 2)^2 - 1$ b $(x - 5)^2 - 28$
c $(x - 4)^2 - 16$ d $(x + 3)^2 - 9$
e $(x - 1)^2 + 6$ f $\left(x+\frac{3}{2}\right)^2 - \frac{17}{4}$

2 a $2(x - 2)^2 - 24$ b $4(x - 1)^2 - 20$

c $3(x+2)^2 - 21$

d $2\left(x+\frac{3}{2}\right)^2 - \frac{25}{2}$

3 a $2\left(x+\frac{3}{4}\right)^2 + \frac{39}{8}$

b $3\left(x-\frac{1}{3}\right)^2 - \frac{1}{3}$

c $5\left(x+\frac{3}{10}\right)^2 - \frac{9}{20}$

d $3\left(x+\frac{5}{6}\right)^2 + \frac{11}{12}$

4 $(5x+3)^2 + 3$

Solving quadratic equations by factorisation

1 a $x = 0$ or $x = -\frac{2}{3}$

b $x = 0$ or $x = \frac{3}{4}$

c $x = -5$ or $x = -2$

d $x = 2$ or $x = 3$

e $x = -1$ or $x = 4$

f $x = -5$ or $x = 2$

g $x = 4$ or $x = 6$

h $x = -6$ or $x = 6$

i $x = -7$ or $x = 4$

j $x = 3$

k $x = -\frac{1}{2}$ or $x = 4$

l $x = -\frac{2}{3}$ or $x = 5$

2 a $x = -2$ or $x = 5$

b $x = -1$ or $x = 3$

c $x = -8$ or $x = 3$

d $x = -6$ or $x = 7$

e $x = -5$ or $x = 5$

f $x = -4$ or $x = 7$

g $x = -3$ or $x = 2\frac{1}{2}$

h $x = -\frac{1}{3}$ or $x = 2$

Solving quadratic equations by completing the square

1 a $x = 2 + \sqrt{7}$ or $x = 2 - \sqrt{7}$

b $x = 5 + \sqrt{21}$ or $x = 5 - \sqrt{21}$

c $x = -4 + \sqrt{21}$ or $x = -4 - \sqrt{21}$

d $x = 1 + \sqrt{7}$ or $x = 1 - \sqrt{7}$

e $x = -2 + \sqrt{6.5}$ or $x = -2 - \sqrt{6.5}$

f $x = \frac{-3+\sqrt{89}}{10}$ or $x = \frac{-3-\sqrt{89}}{10}$

2 a $x = 1 + \sqrt{14}$ or $x = 1 - \sqrt{14}$

b $x = \frac{-3+\sqrt{23}}{2}$ or $x = \frac{-3-\sqrt{23}}{2}$

c $x = \frac{5+\sqrt{13}}{2}$ or $x = \frac{5-\sqrt{13}}{2}$

Solving quadratic equations by using the formula

1 **a** $x = -1 + \frac{\sqrt{3}}{3}$ or $x = -1 - \frac{\sqrt{3}}{3}$ **b** $x = 1 + \frac{3\sqrt{2}}{2}$ or $x = 1 - \frac{3\sqrt{2}}{2}$

2 $x = \frac{7+\sqrt{41}}{2}$ or $x = \frac{7-\sqrt{41}}{2}$

3 $x = \frac{-3+\sqrt{89}}{20}$ or $x = \frac{-3-\sqrt{89}}{20}$

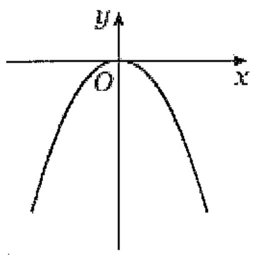
4 **a** $x = \frac{7+\sqrt{17}}{8}$ or $x = \frac{7-\sqrt{17}}{8}$

b $x = -1 + \sqrt{10}$ or $x = -1 - \sqrt{10}$

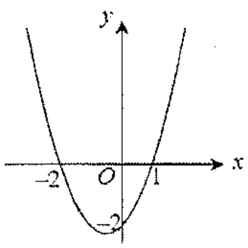
c $x = -1\frac{2}{3}$ or $x = 2$

Sketching quadratic graphs

1



2 **a**



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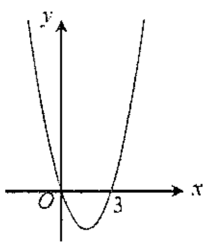

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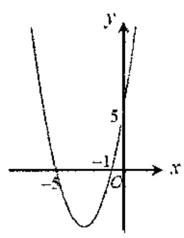
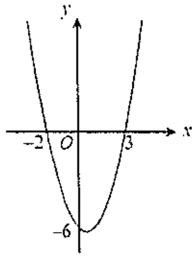


3 **a**



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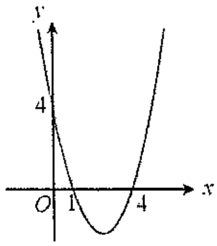

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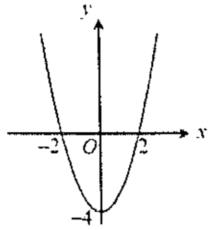

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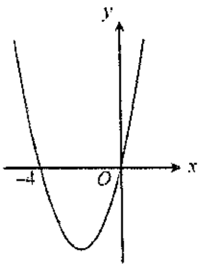

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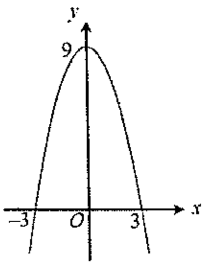

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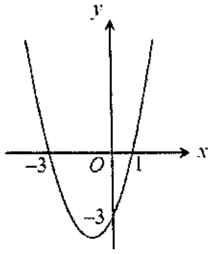

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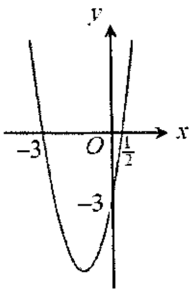
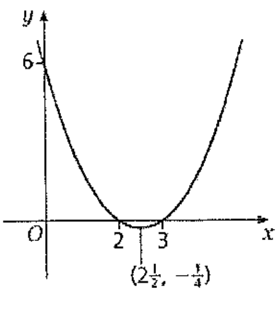


5 **a**



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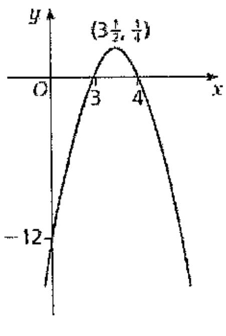

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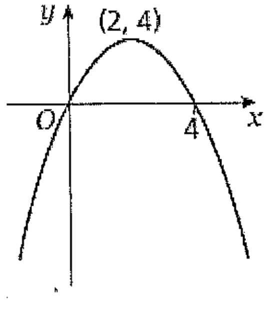

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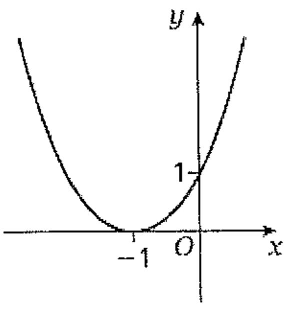


Line of symmetry at $x = -1$.

Solving linear simultaneous equations using the elimination method

1 $x = 1, y = 4$

2 $x = 3, y = -2$

3 $x = 2, y = -5$

4 $x = 3, y = -\frac{1}{2}$

5 $x = 6, y = -1$

6 $x = -2, y = 5$

Solving linear simultaneous equations using the substitution method

1 $x = 9, y = 5$

2 $x = -2, y = -7$

3 $x = \frac{1}{2}, y = 3\frac{1}{2}$

4 $x = \frac{1}{2}, y = 3$

5 $x = -4, y = 5$

6 $x = -2, y = -5$

7 $x = \frac{1}{4}, y = 1\frac{3}{4}$

8 $x = -2, y = 2\frac{1}{2}$

9 $x = -2\frac{1}{2}, y = 5\frac{1}{2}$

Solving linear and quadratic simultaneous equations

1 $x = 1, y = 3$

$x = -\frac{9}{5}, y = -\frac{13}{5}$

2 $x = 2, y = 4$

$x = 4, y = 2$

3 $x = 1, y = -2$

$x = 2, y = -1$

4 $x = 4, y = 1$

$x = \frac{16}{5}, y = \frac{13}{5}$

5 $x = 3, y = 4$

$x = 2, y = 1$

6 $x = 7, y = 2$

$x = -1, y = -6$

7 $x = 0, y = 5$

$x = -5, y = 0$

8 $x = -\frac{8}{3}, y = -\frac{19}{3}$

$x = 3, y = 5$

9 $x = -2, y = -4$

$x = 2, y = 4$

10 $x = \frac{5}{2}, y = 6$

$x = 3, y = 5$

11 $x = \frac{1+\sqrt{5}}{2}, y = \frac{-1+\sqrt{5}}{2}$

$x = \frac{1-\sqrt{5}}{2}, y = \frac{-1-\sqrt{5}}{2}$

12 $x = \frac{-1+\sqrt{7}}{2}, y = \frac{3+\sqrt{7}}{2}$

$x = \frac{-1-\sqrt{7}}{2}, y = \frac{3-\sqrt{7}}{2}$

Solving simultaneous equations graphically

1 a $x = 2, y = 5$
b $x = 2, y = -3$
c $x = -0.5, y = 2.5$

2 a $x = -2, y = 2$
b $x = 0.5, y = 0.5$
c $x = -1, y = -2$

3 a $x = 1, y = 0$ and $x = 4, y = 3$
b $x = -2, y = 7$ and $x = 2, y = -5$
c $x = -2, y = 5$ and $x = -1, y = 4$

4 $x = -3, y = 4$ and $x = 4, y = -3$

5 a i $x = 2.5, y = -2$ and $x = -0.5, y = 4$
ii $x = 2.41, y = -1.83$ and $x = -0.41, y = 3.83$
b Solving algebraically gives the more accurate solutions as the solutions from the graph are only estimates, based on the accuracy of your graph.

Linear inequalities

1 a $x > 4$ b $x \leq 2$ c $x \leq -1$
d $x > -\frac{7}{2}$ e $x \geq 10$ f $x < -15$

2 a $x < -20$ b $x \leq 3.5$ c $x < 4$

3 a $x \leq -4$ b $-1 \leq x < 5$ c $x \leq 1$
d $x < -3$ e $x > 2$ f $x \leq -6$

4 a $t < \frac{5}{2}$ b $n \geq \frac{7}{5}$

5 a $x < -6$ b $x < \frac{3}{2}$

6 $x > 5$ (which also satisfies $x > 3$)

Quadratic inequalities

1 $-7 \leq x \leq 4$

2 $x \leq -2$ or $x \geq 6$

3 $\frac{1}{2} < x < 3$

4 $x < -\frac{3}{2}$ or $x > \frac{1}{2}$

5 $-3 \leq x \leq 4$

6 $-3 \leq x \leq 2$

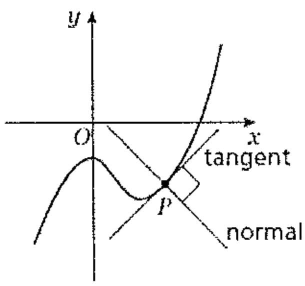
7 $2 < x < 2\frac{1}{2}$

8 $x \leq -\frac{3}{2}$ or $x \geq \frac{5}{3}$

Sketching cubic and reciprocal graphs

1 a i – C
ii – E
iii – B
iv – A
v – F
vi – D

b ii



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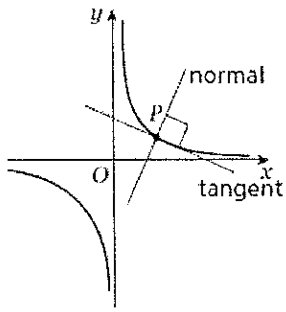

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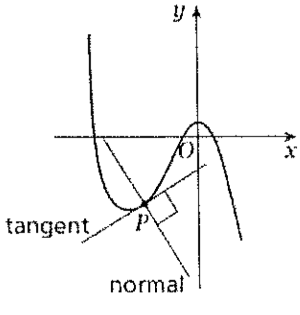

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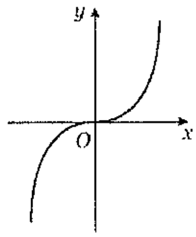

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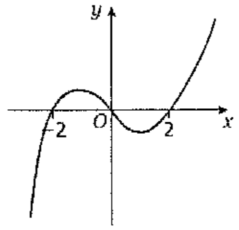

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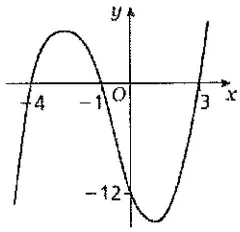

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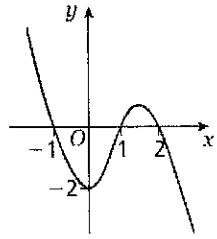

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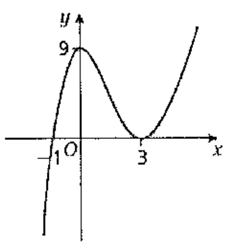

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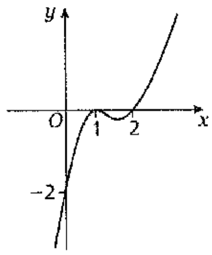

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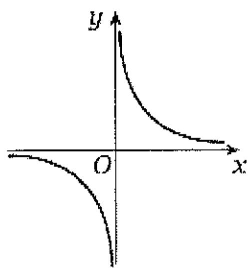

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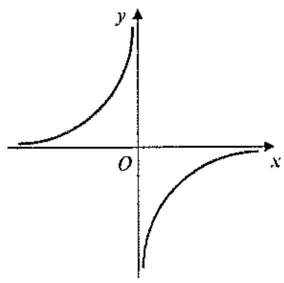

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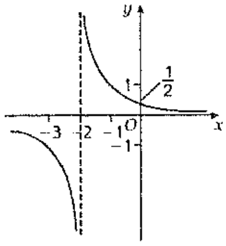

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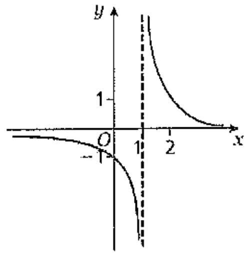

Straight line graphs

1 **a** $m = 3, c = 5$ **b** $m = -\frac{1}{2}, c = -7$

c $m = 2, c = -\frac{3}{2}$ **d** $m = -1, c = 5$

e $m = \frac{2}{3}, c = -\frac{7}{3}$ or $-2\frac{1}{3}$ **f** $m = -5, c = 4$

2

Gradient	y-intercept	Equation of the line
5	0	$y = 5x$
–3	2	$y = -3x + 2$
4	–7	$y = 4x - 7$

3 **a** $x + 2y + 14 = 0$ **b** $2x - y = 0$

c $2x - 3y + 12 = 0$ **d** $6x + 5y + 10 = 0$

4 $y = 4x - 3$

5 $y = -\frac{2}{3}x + 7$

6 **a** $y = 2x - 3$ **b** $y = -\frac{1}{2}x + 6$

c $y = 5x - 2$ **d** $y = -3x + 19$

7 $y = -\frac{3}{2}x + 3$, the gradient is $-\frac{3}{2}$ and the y-intercept is 3.
The line intercepts the axes at (0, 3) and (2, 0).
Students may sketch the line or give coordinates that lie on the line such as $\left(1, \frac{3}{2}\right)$ or $(4, -3)$.

Parallel and perpendicular lines

1	a	$y = 3x - 7$	b	$y = -2x + 5$			
	c	$y = -\frac{1}{2}x$	d	$y = \frac{3}{2}x + 8$			
2		$y = -2x - 7$					
3	a	$y = -\frac{1}{2}x + 2$	b	$y = 3x + 7$			
	c	$y = -4x + 35$	d	$y = \frac{5}{2}x - 8$			
4	a	$y = -\frac{1}{2}x$	b	$y = 2x$			
5	a	Parallel	b	Neither	c	Perpendicular	
	d	Perpendicular	e	Neither	f	Parallel	
6	a	$x + 2y - 4 = 0$	b	$x + 2y + 2 = 0$	c	$y = 2x$	

The cosine rule

1	a	6.46 cm	b	9.26 cm	c	70.8 mm	d	9.70 cm
2	a	22.2°	b	52.9°	c	122.9°	d	93.6°
3	a	13.7 cm	b	76.0°				

The sine rule

1	a	4.33 cm	b	15.0 cm	c	45.2 mm	d	6.39 cm
2	a	42.8°	b	52.8°	c	53.6°	d	28.2°
3	a	8.13 cm	b	32.3°				

Areas of triangles

1 **a** 18.1 cm^2 **b** 18.7 cm^2 **c** 693 mm^2

2 5.10 cm

3 **a** 6.29 cm **b** 84.3° **c** 5.73 cm **d** 58.8°

4 15.3 cm

Rearranging equations

1 $d = \frac{C}{\pi}$ 2 $w = \frac{P-2l}{2}$ 3 $T = \frac{S}{D}$

4 $t = \frac{q-r}{p}$ 5 $t = \frac{2u}{2a-1}$ 6 $x = \frac{V}{a+4}$

7 $y = 2 + 3x$ 8 $a = \frac{3x+1}{x+2}$ 9 $d = \frac{b-c}{x}$

10 $g = \frac{2h+9}{7-h}$ 11 $e = \frac{1}{x+7}$ 12 $x = \frac{4y-3}{2+y}$

13 **a** $r = \sqrt{\frac{A}{\pi}}$ **b** $r = \sqrt[3]{\frac{3V}{4\pi}}$

c $r = \frac{P}{\pi+2}$ **d** $r = \sqrt{\frac{3V}{2\pi h}}$

14 **a** $x = \frac{abz}{cdy}$ **b** $x = \frac{3dz}{4\pi cpy^2}$

15 $\sin B = \frac{b\sin A}{a}$

16 $\cos B = \frac{a^2+c^2-b^2}{2ac}$

17 **a** $x = \frac{q+pt}{q-ps}$ **b** $x = \frac{3py+2pqy}{3p-apq} = \frac{y(3+2q)}{3-aq}$

Area under a graph

1 34 $units^2$

2 149 $units^2$

3 14 $units^2$

4 $25\frac{1}{4}$ $units^2$

5 35 $units^2$

6 42 $units^2$

7 $26\frac{7}{8}$ $units^2$

8 56 $units^2$

9 35 $units^2$

10 $6\frac{1}{4}$ $units^2$