



Stoke Newington School  
& Sixth Form

Name	
------	--

<p>A Level Mathematics Summer Assignment</p>
--



# Maths A Level Summer Assignment & Transition Work

The summer assignment element should take no longer than **2 hours to complete**. Your summer assignment for each course must be submitted in the **relevant first** lesson in September. As a guideline, the transition work element will take 2 hours a week over the summer holiday.

This booklet has been produced to help you prepare for A-Level Maths. It is essential that you begin the course having maintained and developed your skills in the higher-level algebra content from the GCSE syllabus. You also need to understand the volume of work that will be required for A-level Maths. Completion of this booklet will provide evidence that you have the commitment and work ethic required. Additionally, this work will prepare you for the base line test in September.

**INSTRUCTIONS:** there are 2 parts to this booklet.

**Part A: Summer Assignment (submitted in 1st lesson of the year)**

This must be completed in full on **A4 paper** and handed in to your teacher during the first lesson of the year.

You must **show all workings** and set your work out clearly and logically, clearly labelling each question and sub-question. *Do not attempt to complete this on a printout of the assignment-there is not enough room for your full workings and solutions so it must be handed in on A4 paper.*

The assignment will be marked by your teacher. It should take **no more than 2 hours to complete**, once you have done any necessary revision in advance. The transition work in this booklet will prepare you for your summer assignment.

You will also be given a **1-hour baseline test** in the first lesson of the year.

**Part B: Transition Work**

This booklet has been produced by our exam board **Pearson Edexcel** to ensure that all A-Level Maths students across the country begin A level with the algebra skills necessary for the course. It includes all the worksheets that we have advised you to complete during the Summer Term **and more**. You are required to complete **all of this work over the Summer**. If you have already completed some of it during the Summer Term, then you do not need to complete it again, but do use this opportunity to address any gaps. As well as being compulsory, it will help with any necessary preparation for the summer assignment and the baseline test in the first lesson:

- Complete the answers to the exercises on A4 squared paper, with each topic and question number clearly labelled. You must show all working (final answers alone are not accepted).
- If you are stuck, use the Key Points and Examples to remind yourself of how to answer.
- Answers to all of the questions are at given at the end of each section. You should use these answers to correct all of your work in a different coloured pen. If you have a different answer you are expected to make corrections, again showing all of your methods. Correcting your work is a vital skill in A-level mathematics as it contributes to improving your understanding in key areas

To summarise, in September you will be assessed in 3 ways:

1. Part A: Summer Assignment will be formally marked and graded by your teacher.
2. Part B: Transition Work should be self-marked and handed in to your teacher.
3. 1-hour Baseline Test in your first lesson, which will be formally marked and graded by your teacher. Parts A & B of this assignment will help you prepare for this.

# Part A: Summer Assignment (2 hours work)

- 1) Show that  $(x - 3)^2(2x + 5)$  simplifies to  $ax^3 + bx^2 + cx + d$  where  $a, b, c$  and  $d$  are integers to be found.

(3)

- 2) Functions  $f$  and  $g$  are such that  $f(x) = 4x - 5$  and  $g(x) = \frac{12}{2x + 3}$

(a) Find the value of  $g(-5)$ .

(1)

(b) Find  $gf(x)$  in the simplest form.

(2)

© Find the inverse function  $g^{-1}(x)$ .

(2)

- 3) Sketch the graph of  $f(x) = x^2 + 9x + 25$ , writing the coordinates of the turning point and the coordinates of any intercepts with the coordinate axes.

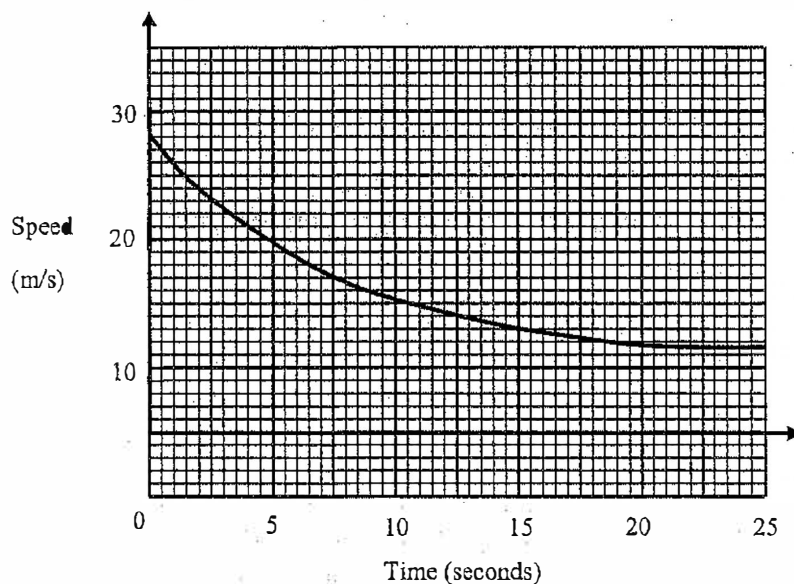
(3)

- 4) By completing the square, write the function  $f(x) = 2x^2 - 3x - 8$  in the form  $f(x) = a(x + b)^2 + c$

where  $a, b$  and  $c$  are rational numbers to be found.

(3)

- 5) The graph shows the speed, in metres per second, of a car as it approaches a set of traffic lights.

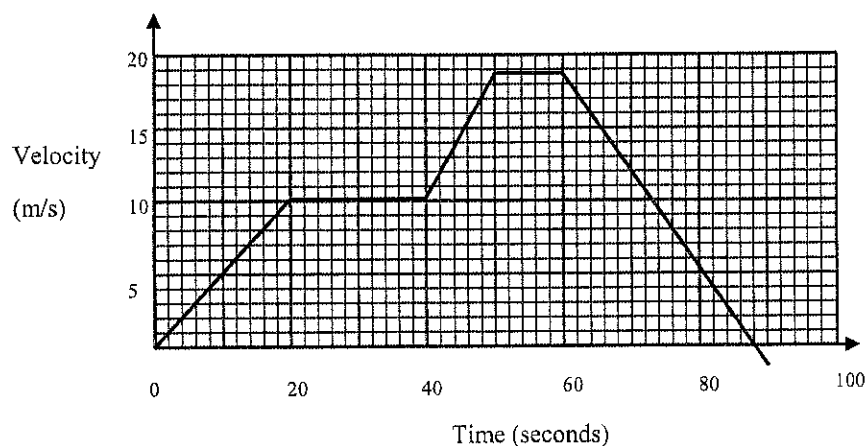


Use the graph to estimate the distance travelled by the car in the first 20 seconds.

(3)

- 6). A computer uses the iteration  $x_{n+1} = \frac{3x_n^2 + 5}{9}$  to find one solution for a quadratic equation.
- (a) What quadratic equation is being solved? (1)
- (b) Using this iterative formula, with  $x_0 = 0$ , find a solution to this quadratic equation to 3d.p. (2)
- 7) Solve  $\frac{x^2 - 16}{3} \geq 2x$  (3)
- 8). If a city's population grows by 20% every year, how many years will it take for this city to double in population? (2)
- 9) The first 5 terms of a quadratic sequence are  
                   -4        -5        -2        5        16  
 Find the expression, in terms of  $n$ , for the  $n$ th term of this sequence. (3)
- 10) The following shows the results of a Higher Education Open Day survey of 100 people.
- 79 wanted to attend Maths.
  - 55 wanted to attend Physics
  - 32 wanted to attend Chemistry
  - 43 wanted to attend both Maths and Physics
  - 16 wanted to attend Physics and Chemistry
  - 18 wanted to attend Maths and Chemistry
  - 6 do not want to attend any of these subjects (they want to choose different subjects altogether)
- Find the probability that a randomly selected person from the survey wants to attend all three of these subjects. (4)
- 11) Using the quadratic formula, find all the solutions to  $2x^2 - x - 5 = 0$ . (2)

- 12). The graph shows the speed of a motorbike over a period of 1.5 minutes.



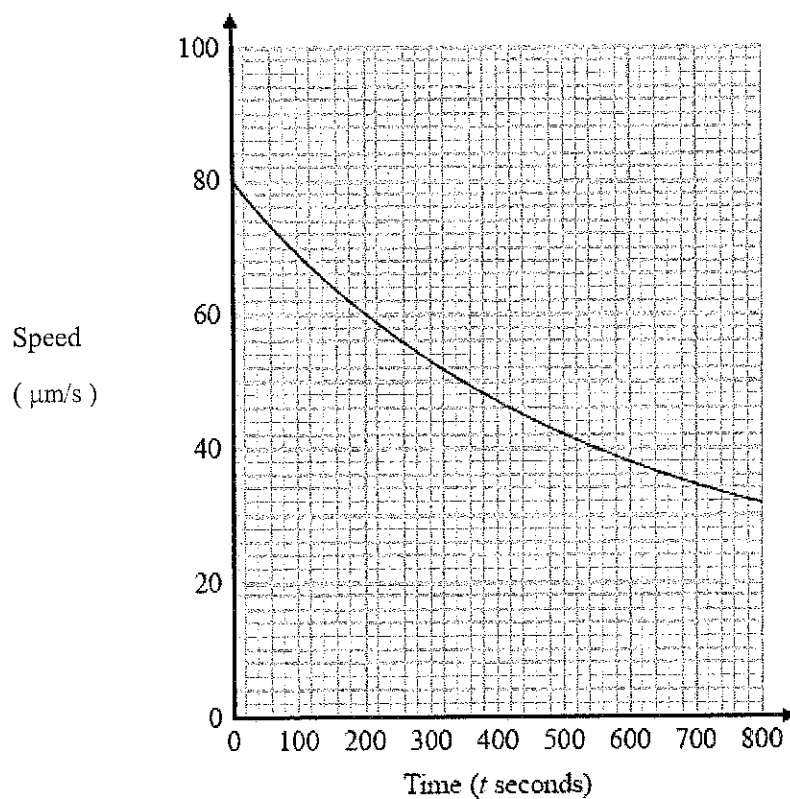
- (a) Between which times was the acceleration the greatest?

(1)

- (b) Find the value of this acceleration.

(2)

- 13) The graph shows the speed, in micrometres per second, of a snail crossing a garden path during a hot summer day.



Work out the average deceleration of the snail in the first 600 seconds.

(2)

14. Show that  $\frac{1}{1 + \frac{1}{\sqrt{2}}}$  can be written as  $2 - \sqrt{2}$

(3)

15.  $ABD$  is a right angled triangle.

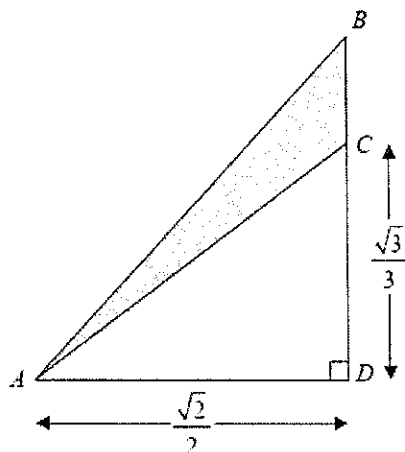


Diagram NOT  
accurately drawn

All measurements are given in centimetres.

$C$  is the point on  $BD$  such that  $CD = \frac{\sqrt{3}}{3}$

$AD = BD = \frac{\sqrt{2}}{2}$

Work out the exact area, in  $\text{cm}^2$ , of the shaded region.

.....  $\text{cm}^2$   
(3)

16. (a) Find the value of  $\sqrt[3]{8 \times 10^6}$

.....  
(1)

- (b) Find the value of  $144^{\frac{1}{2}} \times 64^{-\frac{1}{3}}$

.....  
(2)

- (c) Solve  $3^{2x} = \frac{1}{81}$

$x =$  .....  
(2)

17.

(a) Simplify  $a^4 \times a^3$

.....  
(1)

(b) Simplify  $(b^2)^7$

.....  
(1)

(c) Write down the value of  $3^0$

.....  
(1)

(d) Write down the value of  $4^{-1}$

.....  
(1)



## Part B: Transition Work

### Expanding brackets and simplifying expressions

#### A LEVEL LINKS

**Scheme of work:** 1a. Algebraic expressions – basic algebraic manipulation, indices and surds

#### Key points

- When you expand one set of brackets you must multiply everything inside the bracket by what is outside.
- When you expand two linear expressions, each with two terms of the form  $ax + b$ , where  $a \neq 0$  and  $b \neq 0$ , you create four terms. Two of these can usually be simplified by collecting like terms.

#### Examples

**Example 1** Expand  $4(3x - 2)$

$$4(3x - 2) = 12x - 8$$

Multiply everything inside the bracket by the 4 outside the bracket

**Example 2** Expand and simplify  $3(x + 5) - 4(2x + 3)$

$$\begin{aligned} 3(x + 5) - 4(2x + 3) \\ = 3x + 15 - 8x - 12 \\ = 3 - 5x \end{aligned}$$

- 1 Expand each set of brackets separately by multiplying  $(x + 5)$  by 3 and  $(2x + 3)$  by  $-4$
- 2 Simplify by collecting like terms:  
 $3x - 8x = -5x$  and  $15 - 12 = 3$

**Example 3** Expand and simplify  $(x + 3)(x + 2)$

$$\begin{aligned} (x + 3)(x + 2) \\ = x(x + 2) + 3(x + 2) \\ = x^2 + 2x + 3x + 6 \\ = x^2 + 5x + 6 \end{aligned}$$

- 1 Expand the brackets by multiplying  $(x + 2)$  by  $x$  and  $(x + 2)$  by 3
- 2 Simplify by collecting like terms:  
 $2x + 3x = 5x$

**Example 4** Expand and simplify  $(x - 5)(2x + 3)$

$$\begin{aligned} (x - 5)(2x + 3) \\ = x(2x + 3) - 5(2x + 3) \\ = 2x^2 + 3x - 10x - 15 \\ = 2x^2 - 7x - 15 \end{aligned}$$

- 1 Expand the brackets by multiplying  $(2x + 3)$  by  $x$  and  $(2x + 3)$  by  $-5$
- 2 Simplify by collecting like terms:  
 $3x - 10x = -7x$

## Practice

1 Expand.

a  $3(2x - 1)$

c  $-(3xy - 2y^2)$

b  $-2(5pq + 4q^2)$

2 Expand and simplify.

a  $7(3x + 5) + 6(2x - 8)$

c  $9(3s + 1) - 5(6s - 10)$

b  $8(5p - 2) - 3(4p + 9)$

d  $2(4x - 3) - (3x + 5)$

3 Expand.

a  $3x(4x + 8)$

c  $-2h(6h^2 + 11h - 5)$

b  $4k(5k^2 - 12)$

d  $-3s(4s^2 - 7s + 2)$

4 Expand and simplify.

a  $3(y^2 - 8) - 4(y^2 - 5)$

c  $4p(2p - 1) - 3p(5p - 2)$

b  $2x(x + 5) + 3x(x - 7)$

d  $3b(4b - 3) - b(6b - 9)$

5 Expand  $\frac{1}{2}(2y - 8)$

6 Expand and simplify.

a  $13 - 2(m + 7)$

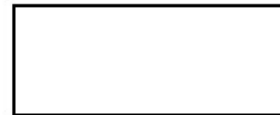
b  $5p(p^2 + 6p) - 9p(2p - 3)$

7 The diagram shows a rectangle.

Write down an expression, in terms of  $x$ , for the area of the rectangle.

Show that the area of the rectangle can be written as  $21x^2 - 35x$

$3x - 5$



$7x$

8 Expand and simplify.

a  $(x + 4)(x + 5)$

c  $(x + 7)(x - 2)$

e  $(2x + 3)(x - 1)$

g  $(5x - 3)(2x - 5)$

i  $(3x + 4y)(5y + 6x)$

k  $(2x - 7)^2$

b  $(x + 7)(x + 3)$

d  $(x + 5)(x - 5)$

f  $(3x - 2)(2x + 1)$

h  $(3x - 2)(7 + 4x)$

j  $(x + 5)^2$

l  $(4x - 3y)^2$

## Extend

9 Expand and simplify  $(x + 3)^2 + (x - 4)^2$

10 Expand and simplify.

a  $\left(x + \frac{1}{x}\right)\left(x - \frac{2}{x}\right)$

b  $\left(x + \frac{1}{x}\right)^2$

### Watch out!

When multiplying (or dividing) positive and negative numbers, if the signs are the same the answer is '+'; if the signs are different the answer is '-'.

# Surds and rationalising the denominator

## A LEVEL LINKS

**Scheme of work:** 1a. Algebraic expressions – basic algebraic manipulation, indices and surds

## Key points

- A surd is the square root of a number that is not a square number, for example  $\sqrt{2}, \sqrt{3}, \sqrt{5}$ , etc.
- Surds can be used to give the exact value for an answer.
- $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
- $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$
- To rationalise the denominator means to remove the surd from the denominator of a fraction.
- To rationalise  $\frac{a}{\sqrt{b}}$  you multiply the numerator and denominator by the surd  $\sqrt{b}$
- To rationalise  $\frac{a}{b + \sqrt{c}}$  you multiply the numerator and denominator by  $b - \sqrt{c}$

## Examples

**Example 1** Simplify  $\sqrt{50}$

$\begin{aligned}\sqrt{50} &= \sqrt{25 \times 2} \\ &= \sqrt{25} \times \sqrt{2} \\ &= 5 \times \sqrt{2} \\ &= 5\sqrt{2}\end{aligned}$	<ol style="list-style-type: none"> <li>1 Choose two numbers that are factors of 50. One of the factors must be a square number</li> <li>2 Use the rule <math>\sqrt{ab} = \sqrt{a} \times \sqrt{b}</math></li> <li>3 Use <math>\sqrt{25} = 5</math></li> </ol>
---	---

**Example 2** Simplify  $\sqrt{147} - 2\sqrt{12}$

$\begin{aligned}\sqrt{147} - 2\sqrt{12} \\ &= \sqrt{49 \times 3} - 2\sqrt{4 \times 3} \\ &= \sqrt{49} \times \sqrt{3} - 2\sqrt{4} \times \sqrt{3} \\ &= 7 \times \sqrt{3} - 2 \times 2 \times \sqrt{3} \\ &= 7\sqrt{3} - 4\sqrt{3} \\ &= 3\sqrt{3}\end{aligned}$	<ol style="list-style-type: none"> <li>1 Simplify <math>\sqrt{147}</math> and <math>2\sqrt{12}</math>. Choose two numbers that are factors of 147 and two numbers that are factors of 12. One of each pair of factors must be a square number</li> <li>2 Use the rule <math>\sqrt{ab} = \sqrt{a} \times \sqrt{b}</math></li> <li>3 Use <math>\sqrt{49} = 7</math> and <math>\sqrt{4} = 2</math></li> <li>4 Collect like terms</li> </ol>
--	--

**Example 3** Simplify  $(\sqrt{7} + \sqrt{2})(\sqrt{7} - \sqrt{2})$

$  \begin{aligned}  &(\sqrt{7} + \sqrt{2})(\sqrt{7} - \sqrt{2}) \\  &= \sqrt{49} - \sqrt{7}\sqrt{2} + \sqrt{2}\sqrt{7} - \sqrt{4} \\  &= 7 - 2 \\  &= 5  \end{aligned}  $	<p><b>1</b> Expand the brackets. A common mistake here is to write <math>(\sqrt{7})^2 = 49</math></p> <p><b>2</b> Collect like terms:  <math>-\sqrt{7}\sqrt{2} + \sqrt{2}\sqrt{7}</math>  <math>= -\sqrt{7}\sqrt{2} + \sqrt{7}\sqrt{2} = 0</math></p>
---	---

**Example 4** Rationalise  $\frac{1}{\sqrt{3}}$

$  \begin{aligned}  \frac{1}{\sqrt{3}} &= \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \\  &= \frac{1 \times \sqrt{3}}{\sqrt{9}} \\  &= \frac{\sqrt{3}}{3}  \end{aligned}  $	<p><b>1</b> Multiply the numerator and denominator by <math>\sqrt{3}</math></p> <p><b>2</b> Use <math>\sqrt{9} = 3</math></p>
---	---

**Example 5** Rationalise and simplify  $\frac{\sqrt{2}}{\sqrt{12}}$

$  \begin{aligned}  \frac{\sqrt{2}}{\sqrt{12}} &= \frac{\sqrt{2}}{\sqrt{12}} \times \frac{\sqrt{12}}{\sqrt{12}} \\  &= \frac{\sqrt{2} \times \sqrt{4 \times 3}}{12} \\  &= \frac{2\sqrt{2}\sqrt{3}}{12} \\  &= \frac{\sqrt{2}\sqrt{3}}{6}  \end{aligned}  $	<p><b>1</b> Multiply the numerator and denominator by <math>\sqrt{12}</math></p> <p><b>2</b> Simplify <math>\sqrt{12}</math> in the numerator. Choose two numbers that are factors of 12. One of the factors must be a square number</p> <p><b>3</b> Use the rule <math>\sqrt{ab} = \sqrt{a} \times \sqrt{b}</math></p> <p><b>4</b> Use <math>\sqrt{4} = 2</math></p> <p><b>5</b> Simplify the fraction:  <math>\frac{2}{12}</math> simplifies to <math>\frac{1}{6}</math></p>
---	--

**Example 6** Rationalise and simplify  $\frac{3}{2+\sqrt{5}}$

$\frac{3}{2+\sqrt{5}} = \frac{3}{2+\sqrt{5}} \times \frac{2-\sqrt{5}}{2-\sqrt{5}}$ $= \frac{3(2-\sqrt{5})}{(2+\sqrt{5})(2-\sqrt{5})}$ $= \frac{6-3\sqrt{5}}{4+2\sqrt{5}-2\sqrt{5}-5}$ $= \frac{6-3\sqrt{5}}{-1}$ $= 3\sqrt{5}-6$	<p><b>1</b> Multiply the numerator and denominator by <math>2-\sqrt{5}</math></p> <p><b>2</b> Expand the brackets</p> <p><b>3</b> Simplify the fraction</p> <p><b>4</b> Divide the numerator by <math>-1</math> Remember to change the sign of all terms when dividing by <math>-1</math></p>
--	---

## Practice

**1** Simplify.

**a**  $\sqrt{45}$

**c**  $\sqrt{48}$

**e**  $\sqrt{300}$

**g**  $\sqrt{72}$

**b**  $\sqrt{125}$

**d**  $\sqrt{175}$

**f**  $\sqrt{28}$

**h**  $\sqrt{162}$

### Hint

One of the two numbers you choose at the start must be a square number.

**2** Simplify.

**a**  $\sqrt{72} + \sqrt{162}$

**c**  $\sqrt{50} - \sqrt{8}$

**e**  $2\sqrt{28} + \sqrt{28}$

**b**  $\sqrt{45} - 2\sqrt{5}$

**d**  $\sqrt{75} - \sqrt{48}$

**f**  $2\sqrt{12} - \sqrt{12} + \sqrt{27}$

### Watch out!

Check you have chosen the highest square number at the start.

**3** Expand and simplify.

**a**  $(\sqrt{2} + \sqrt{3})(\sqrt{2} - \sqrt{3})$

**c**  $(4 - \sqrt{5})(\sqrt{45} + 2)$

**b**  $(3 + \sqrt{3})(5 - \sqrt{12})$

**d**  $(5 + \sqrt{2})(6 - \sqrt{8})$



4 Rationalise and simplify, if possible.

a  $\frac{1}{\sqrt{5}}$

b  $\frac{1}{\sqrt{11}}$

c  $\frac{2}{\sqrt{7}}$

d  $\frac{2}{\sqrt{8}}$

e  $\frac{2}{\sqrt{2}}$

f  $\frac{5}{\sqrt{5}}$

g  $\frac{\sqrt{8}}{\sqrt{24}}$

h  $\frac{\sqrt{5}}{\sqrt{45}}$

5 Rationalise and simplify.

a  $\frac{1}{3-\sqrt{5}}$

b  $\frac{2}{4+\sqrt{3}}$

c  $\frac{6}{5-\sqrt{2}}$

## Extend

6 Expand and simplify  $(\sqrt{x} + \sqrt{y})(\sqrt{x} - \sqrt{y})$

7 Rationalise and simplify, if possible.

a  $\frac{1}{\sqrt{9}-\sqrt{8}}$

b  $\frac{1}{\sqrt{x}-\sqrt{y}}$

# Rules of indices

## A LEVEL LINKS

**Scheme of work:** 1a. Algebraic expressions – basic algebraic manipulation, indices and surds

## Key points

- $a^m \times a^n = a^{m+n}$
- $\frac{a^m}{a^n} = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $a^0 = 1$
- $a^{\frac{1}{n}} = \sqrt[n]{a}$  i.e. the  $n$ th root of  $a$
- $a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$
- $a^{-m} = \frac{1}{a^m}$
- The square root of a number produces two solutions, e.g.  $\sqrt{16} = \pm 4$ .

## Examples

**Example 1** Evaluate  $10^0$

$10^0 = 1$	Any value raised to the power of zero is equal to 1
------------	---

**Example 2** Evaluate  $9^{\frac{1}{2}}$

$9^{\frac{1}{2}} = \sqrt{9}$ $= 3$	Use the rule $a^{\frac{1}{n}} = \sqrt[n]{a}$
------------------------------------	--

**Example 3** Evaluate  $27^{\frac{2}{3}}$

$27^{\frac{2}{3}} = \left(\sqrt[3]{27}\right)^2$ $= 3^2$ $= 9$	<p><b>1</b> Use the rule <math>a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m</math></p> <p><b>2</b> Use <math>\sqrt[3]{27} = 3</math></p>
--	--

**Example 4** Evaluate  $4^{-2}$

$4^{-2} = \frac{1}{4^2}$ $= \frac{1}{16}$	<p><b>1</b> Use the rule <math>a^{-m} = \frac{1}{a^m}</math></p> <p><b>2</b> Use <math>4^2 = 16</math></p>
---	--

**Example 5** Simplify  $\frac{6x^5}{2x^2}$

$\frac{6x^5}{2x^2} = 3x^3$	<p><math>6 \div 2 = 3</math> and use the rule <math>\frac{a^m}{a^n} = a^{m-n}</math> to</p> <p>give <math>\frac{x^5}{x^2} = x^{5-2} = x^3</math></p>
----------------------------	--

**Example 6** Simplify  $\frac{x^3 \times x^5}{x^4}$

$\frac{x^3 \times x^5}{x^4} = \frac{x^{3+5}}{x^4} = \frac{x^8}{x^4}$ $= x^{8-4} = x^4$	<p><b>1</b> Use the rule <math>a^m \times a^n = a^{m+n}</math></p> <p><b>2</b> Use the rule <math>\frac{a^m}{a^n} = a^{m-n}</math></p>
--	--

**Example 7** Write  $\frac{1}{3x}$  as a single power of  $x$

$\frac{1}{3x} = \frac{1}{3} x^{-1}$	<p>Use the rule <math>\frac{1}{a^m} = a^{-m}</math>, note that the</p> <p>fraction <math>\frac{1}{3}</math> remains unchanged</p>
-------------------------------------	---

**Example 8** Write  $\frac{4}{\sqrt{x}}$  as a single power of  $x$

$\frac{4}{\sqrt{x}} = \frac{4}{x^{\frac{1}{2}}}$ $= 4x^{-\frac{1}{2}}$	<p><b>1</b> Use the rule <math>a^{\frac{1}{n}} = \sqrt[n]{a}</math></p> <p><b>2</b> Use the rule <math>\frac{1}{a^m} = a^{-m}</math></p>
--	--





## Practice

1 Evaluate.

a  $14^0$

b  $3^0$

c  $5^0$

d  $x^0$

2 Evaluate.

a  $49^{\frac{1}{2}}$

b  $64^{\frac{1}{3}}$

c  $125^{\frac{1}{3}}$

d  $16^{\frac{1}{4}}$

3 Evaluate.

a  $25^{\frac{3}{2}}$

b  $8^{\frac{5}{3}}$

c  $49^{\frac{3}{2}}$

d  $16^{\frac{3}{4}}$

4 Evaluate.

a  $5^{-2}$

b  $4^{-3}$

c  $2^{-5}$

d  $6^{-2}$

5 Simplify.

a  $\frac{3x^2 \times x^3}{2x^2}$

b  $\frac{10x^5}{2x^2 \times x}$

c  $\frac{3x \times 2x^3}{2x^3}$

d  $\frac{7x^3y^2}{14x^5y}$

e  $\frac{y^2}{y^{\frac{1}{2}} \times y}$

f  $\frac{c^{\frac{1}{2}}}{c^2 \times c^{\frac{3}{2}}}$

g  $\frac{(2x^2)^3}{4x^0}$

h  $\frac{x^{\frac{1}{2}} \times x^{\frac{3}{2}}}{x^{-2} \times x^3}$

### Watch out!

Remember that any value raised to the power of zero is 1. This is the rule  $a^0 = 1$ .

6 Evaluate.

a  $4^{-\frac{1}{2}}$

b  $27^{-\frac{2}{3}}$

c  $9^{-\frac{1}{2}} \times 2^3$

d  $16^{\frac{1}{4}} \times 2^{-3}$

e  $\left(\frac{9}{16}\right)^{-\frac{1}{2}}$

f  $\left(\frac{27}{64}\right)^{-\frac{2}{3}}$

7 Write the following as a single power of  $x$ .

a  $\frac{1}{x}$

b  $\frac{1}{x^7}$

c  $\sqrt[4]{x}$

d  $\sqrt[5]{x^2}$

e  $\frac{1}{\sqrt[3]{x}}$

f  $\frac{1}{\sqrt[3]{x^2}}$



**8** Write the following without negative or fractional powers.

**a**  $x^{-3}$

**b**  $x^0$

**c**  $x^{\frac{1}{5}}$

**d**  $x^{\frac{2}{5}}$

**e**  $x^{-\frac{1}{2}}$

**f**  $x^{-\frac{3}{4}}$

**9** Write the following in the form  $ax^n$ .

**a**  $5\sqrt{x}$

**b**  $\frac{2}{x^3}$

**c**  $\frac{1}{3x^4}$

**d**  $\frac{2}{\sqrt{x}}$

**e**  $\frac{4}{\sqrt[3]{x}}$

**f**  $3$

## Extend

**10** Write as sums of powers of  $x$ .

**a**  $\frac{x^5 + 1}{x^2}$

**b**  $x^2\left(x + \frac{1}{x}\right)$

**c**  $x^{-4}\left(x^2 + \frac{1}{x^3}\right)$

# Factorising expressions

## A LEVEL LINKS

**Scheme of work:** 1b. Quadratic functions – factorising, solving, graphs and the discriminants

## Key points

- Factorising an expression is the opposite of expanding the brackets.
- A quadratic expression is in the form  $ax^2 + bx + c$ , where  $a \neq 0$ .
- To factorise a quadratic equation find two numbers whose sum is  $b$  and whose product is  $ac$ .
- An expression in the form  $x^2 - y^2$  is called the difference of two squares. It factorises to  $(x - y)(x + y)$ .

## Examples

**Example 1** Factorise  $15x^2y^3 + 9x^4y$

$$15x^2y^3 + 9x^4y = 3x^2y(5y^2 + 3x^2)$$

The highest common factor is  $3x^2y$ .  
So take  $3x^2y$  outside the brackets and then divide each term by  $3x^2y$  to find the terms in the brackets

**Example 2** Factorise  $4x^2 - 25y^2$

$$4x^2 - 25y^2 = (2x + 5y)(2x - 5y)$$

This is the difference of two squares as the two terms can be written as  $(2x)^2$  and  $(5y)^2$

**Example 3** Factorise  $x^2 + 3x - 10$

$$b = 3, ac = -10$$

$$\text{So } x^2 + 3x - 10 = x^2 + 5x - 2x - 10$$

$$= x(x + 5) - 2(x + 5)$$

$$= (x + 5)(x - 2)$$

- 1 Work out the two factors of  $ac = -10$  which add to give  $b = 3$  (5 and -2)
- 2 Rewrite the  $b$  term ( $3x$ ) using these two factors
- 3 Factorise the first two terms and the last two terms
- 4  $(x + 5)$  is a factor of both terms

**Example 4** Factorise  $6x^2 - 11x - 10$

$b = -11, ac = -60$  So $6x^2 - 11x - 10 = 6x^2 - 15x + 4x - 10$ $= 3x(2x - 5) + 2(2x - 5)$ $= (2x - 5)(3x + 2)$	<ol style="list-style-type: none"> <li>1 Work out the two factors of <math>ac = -60</math> which add to give <math>b = -11</math> (-15 and 4)</li> <li>2 Rewrite the <math>b</math> term (<math>-11x</math>) using these two factors</li> <li>3 Factorise the first two terms and the last two terms</li> <li>4 <math>(2x - 5)</math> is a factor of both terms</li> </ol>
---	--

**Example 5** Simplify  $\frac{x^2 - 4x - 21}{2x^2 + 9x + 9}$

$\frac{x^2 - 4x - 21}{2x^2 + 9x + 9}$  For the numerator: $b = -4, ac = -21$  So $x^2 - 4x - 21 = x^2 - 7x + 3x - 21$ $= x(x - 7) + 3(x - 7)$ $= (x - 7)(x + 3)$  For the denominator: $b = 9, ac = 18$  So $2x^2 + 9x + 9 = 2x^2 + 6x + 3x + 9$ $= 2x(x + 3) + 3(x + 3)$ $= (x + 3)(2x + 3)$  So $\frac{x^2 - 4x - 21}{2x^2 + 9x + 9} = \frac{(x - 7)(x + 3)}{(x + 3)(2x + 3)}$ $= \frac{x - 7}{2x + 3}$	<ol style="list-style-type: none"> <li>1 Factorise the numerator and the denominator</li> <li>2 Work out the two factors of <math>ac = -21</math> which add to give <math>b = -4</math> (-7 and 3)</li> <li>3 Rewrite the <math>b</math> term (<math>-4x</math>) using these two factors</li> <li>4 Factorise the first two terms and the last two terms</li> <li>5 <math>(x - 7)</math> is a factor of both terms</li> <li>6 Work out the two factors of <math>ac = 18</math> which add to give <math>b = 9</math> (6 and 3)</li> <li>7 Rewrite the <math>b</math> term (<math>9x</math>) using these two factors</li> <li>8 Factorise the first two terms and the last two terms</li> <li>9 <math>(x + 3)</math> is a factor of both terms</li> <li>10 <math>(x + 3)</math> is a factor of both the numerator and denominator so cancels out as a value divided by itself is 1</li> </ol>
---	---

## Practice

1 Factorise.

a  $6x^4y^3 - 10x^3y^4$

c  $25x^2y^2 - 10x^3y^2 + 15x^2y^3$

b  $21a^3b^5 + 35a^5b^2$

2 Factorise

a  $x^2 + 7x + 12$

c  $x^2 - 11x + 30$

e  $x^2 - 7x - 18$

g  $x^2 - 3x - 40$

b  $x^2 + 5x - 14$

d  $x^2 - 5x - 24$

f  $x^2 + x - 20$

h  $x^2 + 3x - 28$

3 Factorise

a  $36x^2 - 49y^2$

c  $18a^2 - 200b^2c^2$

b  $4x^2 - 81y^2$

4 Factorise

a  $2x^2 + x - 3$

c  $2x^2 + 7x + 3$

e  $10x^2 + 21x + 9$

b  $6x^2 + 17x + 5$

d  $9x^2 - 15x + 4$

f  $12x^2 - 38x + 20$

5 Simplify the algebraic fractions.

a  $\frac{2x^2 + 4x}{x^2 - x}$

c  $\frac{x^2 - 2x - 8}{x^2 - 4x}$

e  $\frac{x^2 - x - 12}{x^2 - 4x}$

b  $\frac{x^2 + 3x}{x^2 + 2x - 3}$

d  $\frac{x^2 - 5x}{x^2 - 25}$

f  $\frac{2x^2 + 14x}{2x^2 + 4x - 70}$

6 Simplify

a  $\frac{9x^2 - 16}{3x^2 + 17x - 28}$

c  $\frac{4 - 25x^2}{10x^2 - 11x - 6}$

b  $\frac{2x^2 - 7x - 15}{3x^2 - 17x + 10}$

d  $\frac{6x^2 - x - 1}{2x^2 + 7x - 4}$

### Hint

Take the highest  
common factor  
outside the bracket.

## Extend

7 Simplify  $\sqrt{x^2 + 10x + 25}$

8 Simplify  $\frac{(x+2)^2 + 3(x+2)^2}{x^2 - 4}$

# Completing the square

## A LEVEL LINKS

**Scheme of work:** 1b. Quadratic functions – factorising, solving, graphs and the discriminants

## Key points

- Completing the square for a quadratic rearranges  $ax^2 + bx + c$  into the form  $p(x + q)^2 + r$
- If  $a \neq 1$ , then factorise using  $a$  as a common factor.

## Examples

**Example 1** Complete the square for the quadratic expression  $x^2 + 6x - 2$

$x^2 + 6x - 2$ $= (x + 3)^2 - 9 - 2$ $= (x + 3)^2 - 11$	<p><b>1</b> Write <math>x^2 + bx + c</math> in the form <math>\left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c</math></p> <p><b>2</b> Simplify</p>
---	---

**Example 2** Write  $2x^2 - 5x + 1$  in the form  $p(x + q)^2 + r$

$2x^2 - 5x + 1$ $= 2\left(x^2 - \frac{5}{2}x\right) + 1$ $= 2\left[\left(x - \frac{5}{4}\right)^2 - \left(\frac{5}{4}\right)^2\right] + 1$ $= 2\left(x - \frac{5}{4}\right)^2 - \frac{25}{8} + 1$ $= 2\left(x - \frac{5}{4}\right)^2 - \frac{17}{8}$	<p><b>1</b> Before completing the square write <math>ax^2 + bx + c</math> in the form <math>a\left(x^2 + \frac{b}{a}x\right) + c</math></p> <p><b>2</b> Now complete the square by writing <math>x^2 - \frac{5}{2}x</math> in the form <math>\left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2</math></p> <p><b>3</b> Expand the square brackets – don't forget to multiply <math>\left(\frac{5}{4}\right)^2</math> by the factor of 2</p> <p><b>4</b> Simplify</p>
--	---



## Practice

1 Write the following quadratic expressions in the form  $(x + p)^2 + q$

**a**  $x^2 + 4x + 3$

**b**  $x^2 - 10x - 3$

**c**  $x^2 - 8x$

**d**  $x^2 + 6x$

**e**  $x^2 - 2x + 7$

**f**  $x^2 + 3x - 2$

2 Write the following quadratic expressions in the form  $p(x + q)^2 + r$

**a**  $2x^2 - 8x - 16$

**b**  $4x^2 - 8x - 16$

**c**  $3x^2 + 12x - 9$

**d**  $2x^2 + 6x - 8$

3 Complete the square.

**a**  $2x^2 + 3x + 6$

**b**  $3x^2 - 2x$

**c**  $5x^2 + 3x$

**d**  $3x^2 + 5x + 3$

## Extend

4 Write  $(25x^2 + 30x + 12)$  in the form  $(ax + b)^2 + c$ .

# Solving linear simultaneous equations using the elimination method

## A LEVEL LINKS

Scheme of work: 1c. Equations – quadratic/linear simultaneous

## Key points

- Two equations are simultaneous when they are both true at the same time.
- Solving simultaneous linear equations in two unknowns involves finding the value of each unknown which works for both equations.
- Make sure that the coefficient of one of the unknowns is the same in both equations.
- Eliminate this equal unknown by either subtracting or adding the two equations.

## Examples

**Example 1** Solve the simultaneous equations  $3x + y = 5$  and  $x + y = 1$

$\begin{array}{r} 3x + y = 5 \\ - \quad x + y = 1 \\ \hline 2x \quad = 4 \end{array}$ <p>So <math>x = 2</math></p> <p>Using <math>x + y = 1</math>  <math>2 + y = 1</math>          So <math>y = -1</math></p> <p>Check:          equation 1: <math>3 \times 2 + (-1) = 5</math> YES          equation 2: <math>2 + (-1) = 1</math> YES</p>	<p><b>1</b> Subtract the second equation from the first equation to eliminate the <math>y</math> term.</p> <p><b>2</b> To find the value of <math>y</math>, substitute <math>x = 2</math> into one of the original equations.</p> <p><b>3</b> Substitute the values of <math>x</math> and <math>y</math> into both equations to check your answers.</p>
---	---

**Example 2** Solve  $x + 2y = 13$  and  $5x - 2y = 5$  simultaneously.

$\begin{array}{r} x + 2y = 13 \\ + \quad 5x - 2y = 5 \\ \hline 6x \quad = 18 \end{array}$ <p>So <math>x = 3</math></p> <p>Using <math>x + 2y = 13</math>  <math>3 + 2y = 13</math>          So <math>y = 5</math></p> <p>Check:          equation 1: <math>3 + 2 \times 5 = 13</math> YES          equation 2: <math>5 \times 3 - 2 \times 5 = 5</math> YES</p>	<p><b>1</b> Add the two equations together to eliminate the <math>y</math> term.</p> <p><b>2</b> To find the value of <math>y</math>, substitute <math>x = 3</math> into one of the original equations.</p> <p><b>3</b> Substitute the values of <math>x</math> and <math>y</math> into both equations to check your answers.</p>
---	---



**Example 3** Solve  $2x + 3y = 2$  and  $5x + 4y = 12$  simultaneously.

$(2x + 3y = 2) \times 4 \rightarrow 8x + 12y = 8$ $(5x + 4y = 12) \times 3 \rightarrow 15x + 12y = 36$ $\underline{7x = 28}$  So $x = 4$  Using $2x + 3y = 2$ $2 \times 4 + 3y = 2$ So $y = -2$  Check: equation 1: $2 \times 4 + 3 \times (-2) = 2$ YES equation 2: $5 \times 4 + 4 \times (-2) = 12$ YES	<b>1</b> Multiply the first equation by 4 and the second equation by 3 to make the coefficient of $y$ the same for both equations. Then subtract the first equation from the second equation to eliminate the $y$ term.  <b>2</b> To find the value of $y$ , substitute $x = 4$ into one of the original equations.  <b>3</b> Substitute the values of $x$ and $y$ into both equations to check your answers.
--	---

## Practice

Solve these simultaneous equations.

**1**  $4x + y = 8$   
 $x + y = 5$

**2**  $3x + y = 7$   
 $3x + 2y = 5$

**3**  $4x + y = 3$   
 $3x - y = 11$

**4**  $3x + 4y = 7$   
 $x - 4y = 5$

**5**  $2x + y = 11$   
 $x - 3y = 9$

**6**  $2x + 3y = 11$   
 $3x + 2y = 4$

# Solving linear simultaneous equations using the substitution method

## A LEVEL LINKS

**Scheme of work:** 1c. Equations – quadratic/linear simultaneous

**Textbook:** Pure Year 1, 3.1 Linear simultaneous equations

## Key points

- The substitution method is the method most commonly used for A level. This is because it is the method used to solve linear and quadratic simultaneous equations.

## Examples

**Example 4** Solve the simultaneous equations  $y = 2x + 1$  and  $5x + 3y = 14$

$5x + 3(2x + 1) = 14$ $5x + 6x + 3 = 14$ $11x + 3 = 14$ $11x = 11$ $\text{So } x = 1$  Using $y = 2x + 1$ $y = 2 \times 1 + 1$ $\text{So } y = 3$  Check: equation 1: $3 = 2 \times 1 + 1$ YES equation 2: $5 \times 1 + 3 \times 3 = 14$ YES	<ol style="list-style-type: none"> <li>1 Substitute <math>2x + 1</math> for <math>y</math> into the second equation.</li> <li>2 Expand the brackets and simplify.</li> <li>3 Work out the value of <math>x</math>.</li> <li>4 To find the value of <math>y</math>, substitute <math>x = 1</math> into one of the original equations.</li> <li>5 Substitute the values of <math>x</math> and <math>y</math> into both equations to check your answers.</li> </ol>
---	--

**Example 5** Solve  $2x - y = 16$  and  $4x + 3y = -3$  simultaneously.

$y = 2x - 16$ $4x + 3(2x - 16) = -3$ $4x + 6x - 48 = -3$ $10x - 48 = -3$ $10x = 45$ $\text{So } x = 4\frac{1}{2}$  Using $y = 2x - 16$ $y = 2 \times 4\frac{1}{2} - 16$ $\text{So } y = -7$  Check: equation 1: $2 \times 4\frac{1}{2} - (-7) = 16$ YES equation 2: $4 \times 4\frac{1}{2} + 3 \times (-7) = -3$ YES	<ol style="list-style-type: none"> <li>1 Rearrange the first equation.</li> <li>2 Substitute <math>2x - 16</math> for <math>y</math> into the second equation.</li> <li>3 Expand the brackets and simplify.</li> <li>4 Work out the value of <math>x</math>.</li> <li>5 To find the value of <math>y</math>, substitute <math>x = 4\frac{1}{2}</math> into one of the original equations.</li> <li>6 Substitute the values of <math>x</math> and <math>y</math> into both equations to check your answers.</li> </ol>
---	---

## Practice

Solve these simultaneous equations.

**7**  $y = x - 4$   
 $2x + 5y = 43$

**8**  $y = 2x - 3$   
 $5x - 3y = 11$

**9**  $2y = 4x + 5$   
 $9x + 5y = 22$

**10**  $2x = y - 2$   
 $8x - 5y = -11$

**11**  $3x + 4y = 8$   
 $2x - y = -13$

**12**  $3y = 4x - 7$   
 $2y = 3x - 4$

**13**  $3x = y - 1$   
 $2y - 2x = 3$

**14**  $3x + 2y + 1 = 0$   
 $4y = 8 - x$

## Extend

**15** Solve the simultaneous equations  $3x + 5y - 20 = 0$  and  $2(x + y) = \frac{3(y - x)}{4}$ .

# Pythagoras' theorem

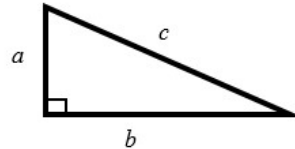
## A LEVEL LINKS

**Scheme of work:** 2a. Straight-line graphs, parallel/perpendicular, length and area problems

## Key points

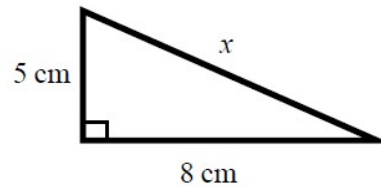
- In a right-angled triangle the longest side is called the hypotenuse.
- Pythagoras' theorem states that for a right-angled triangle the square of the hypotenuse is equal to the sum of the squares of the other two sides.

$$c^2 = a^2 + b^2$$

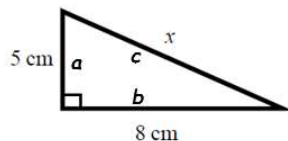


## Examples

**Example 1** Calculate the length of the hypotenuse.  
Give your answer to 3 significant figures.



$$c^2 = a^2 + b^2$$



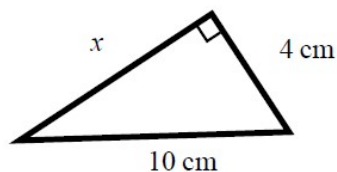
$$\begin{aligned} x^2 &= 5^2 + 8^2 \\ x^2 &= 25 + 64 \\ x^2 &= 89 \\ x &= \sqrt{89} \end{aligned}$$

$$\begin{aligned} x &= 9.433\,981\,13... \\ x &= 9.43 \text{ cm} \end{aligned}$$

- 1 Always start by stating the formula for Pythagoras' theorem and labelling the hypotenuse  $c$  and the other two sides  $a$  and  $b$ .
- 2 Substitute the values of  $a$ ,  $b$  and  $c$  into the formula for Pythagoras' theorem.
- 3 Use a calculator to find the square root.
- 4 Round your answer to 3 significant figures and write the units with your answer.



**Example 2** Calculate the length  $x$ .  
Give your answer in surd form.



$$c^2 = a^2 + b^2$$

$$10^2 = x^2 + 4^2$$

$$100 = x^2 + 16$$

$$x^2 = 84$$

$$x = \sqrt{84}$$

$$x = 2\sqrt{21} \text{ cm}$$

**1** Always start by stating the formula for Pythagoras' theorem.

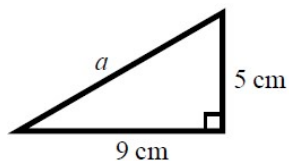
**2** Substitute the values of  $a$ ,  $b$  and  $c$  into the formula for Pythagoras' theorem.

**3** Simplify the surd where possible and write the units in your answer.

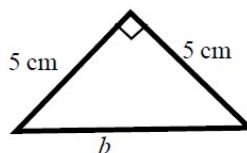
## Practice

**1** Work out the length of the unknown side in each triangle.  
Give your answers correct to 3 significant figures.

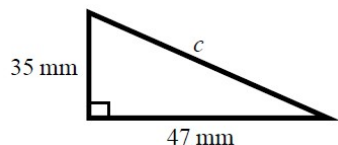
**a**



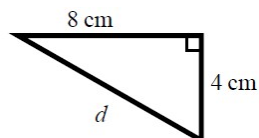
**b**



**c**

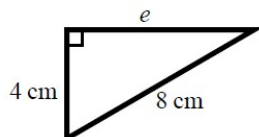


**d**

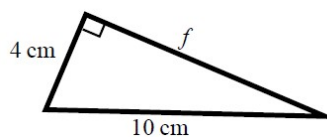


**2** Work out the length of the unknown side in each triangle.  
Give your answers in surd form.

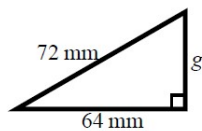
**a**



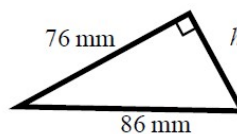
**b**



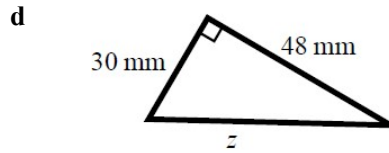
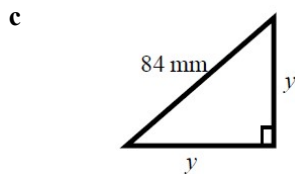
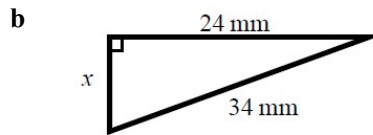
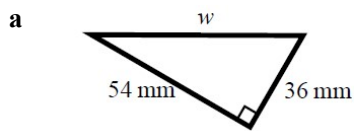
**c**



**d**



- 3 Work out the length of the unknown side in each triangle.  
Give your answers in surd form.



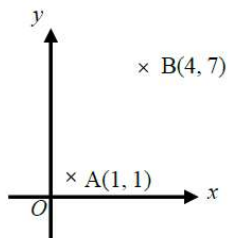
- 4 A rectangle has length 84 mm and width 45 mm.  
Calculate the length of the diagonal of the rectangle.  
Give your answer correct to 3 significant figures.

**Hint**

Draw a sketch of the rectangle.

## Extend

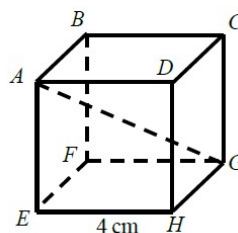
- 5 A yacht is 40 km due North of a lighthouse.  
A rescue boat is 50 km due East of the same lighthouse.  
Work out the distance between the yacht and the rescue boat.  
Give your answer correct to 3 significant figures.
- 6 Points A and B are shown on the diagram.  
Work out the length of the line AB.  
Give your answer in surd form.



**Hint**

Draw a diagram using  
the information given  
in the question.

- 7 A cube has length 4 cm.  
Work out the length of the diagonal AG.  
Give your answer in surd form.



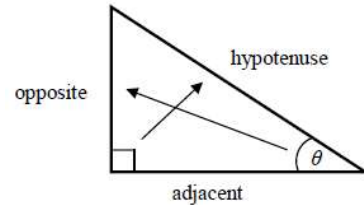
# Trigonometry in right-angled triangles

## A LEVEL LINKS

Scheme of work: 4a. Trigonometric ratios and graphs

## Key points

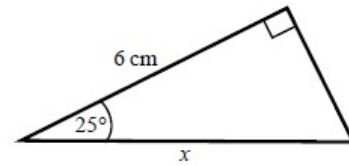
- In a right-angled triangle:
  - the side opposite the right angle is called the hypotenuse
  - the side opposite the angle  $\theta$  is called the opposite
  - the side next to the angle  $\theta$  is called the adjacent.
- In a right-angled triangle:
  - the ratio of the opposite side to the hypotenuse is the sine of angle  $\theta$ ,  $\sin \theta = \frac{\text{opp}}{\text{hyp}}$
  - the ratio of the adjacent side to the hypotenuse is the cosine of angle  $\theta$ ,  $\cos \theta = \frac{\text{adj}}{\text{hyp}}$
  - the ratio of the opposite side to the adjacent side is the tangent of angle  $\theta$ ,  $\tan \theta = \frac{\text{opp}}{\text{adj}}$
- If the lengths of two sides of a right-angled triangle are given, you can find a missing angle using the inverse trigonometric functions:  $\sin^{-1}$ ,  $\cos^{-1}$ ,  $\tan^{-1}$ .
- The sine, cosine and tangent of some angles may be written exactly.

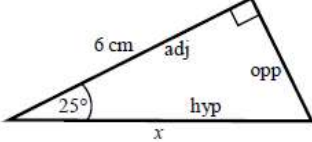


	0	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	

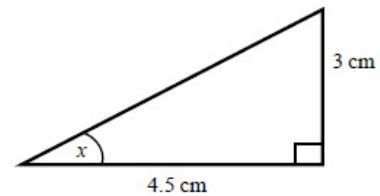
## Examples

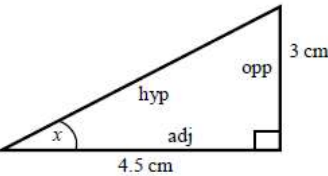
**Example 1** Calculate the length of side  $x$ .  
Give your answer correct to 3 significant figures.



 $\cos \theta = \frac{\text{adj}}{\text{hyp}}$ $\cos 25^\circ = \frac{6}{x}$ $x = \frac{6}{\cos 25^\circ}$ $x = 6.620\,267\,5\dots$ $x = 6.62 \text{ cm}$	<ol style="list-style-type: none"> <li>1 Always start by labelling the sides.</li> <li>2 You are given the adjacent and the hypotenuse so use the cosine ratio.</li> <li>3 Substitute the sides and angle into the cosine ratio.</li> <li>4 Rearrange to make <math>x</math> the subject.</li> <li>5 Use your calculator to work out <math>6 \div \cos 25^\circ</math>.</li> <li>6 Round your answer to 3 significant figures and write the units in your answer.</li> </ol>
--	--

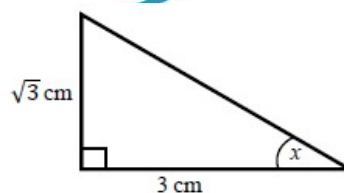
**Example 2** Calculate the size of angle  $x$ .  
Give your answer correct to 3 significant figures.

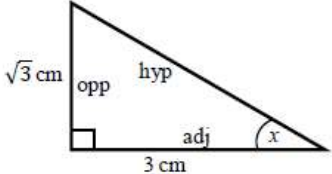


 $\tan \theta = \frac{\text{opp}}{\text{adj}}$ $\tan x = \frac{3}{4.5}$ $x = \tan^{-1} \left( \frac{3}{4.5} \right)$ $x = 33.690\,067\,5\dots$ $x = 33.7^\circ$	<ol style="list-style-type: none"> <li>1 Always start by labelling the sides.</li> <li>2 You are given the opposite and the adjacent so use the tangent ratio.</li> <li>3 Substitute the sides and angle into the tangent ratio.</li> <li>4 Use <math>\tan^{-1}</math> to find the angle.</li> <li>5 Use your calculator to work out <math>\tan^{-1}(3 \div 4.5)</math>.</li> <li>6 Round your answer to 3 significant figures and write the units in your answer.</li> </ol>
--	---



**Example 3** Calculate the exact size of angle  $x$ .

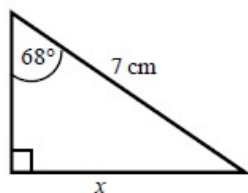


 $\tan \theta = \frac{\text{opp}}{\text{adj}}$ $\tan x = \frac{\sqrt{3}}{3}$ $x = 30^\circ$	<ol style="list-style-type: none"> <li>1 Always start by labelling the sides.</li> <li>2 You are given the opposite and the adjacent so use the tangent ratio.</li> <li>3 Substitute the sides and angle into the tangent ratio.</li> <li>4 Use the table from the key points to find the angle.</li> </ol>
--	---

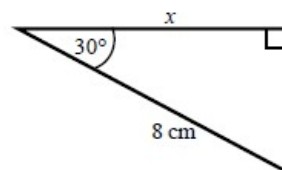
## Practice

1 Calculate the length of the unknown side in each triangle. Give your answers correct to 3 significant figures.

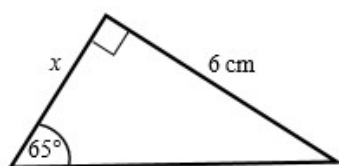
a



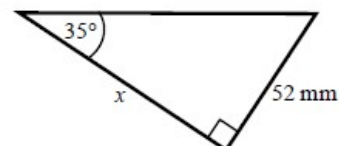
b



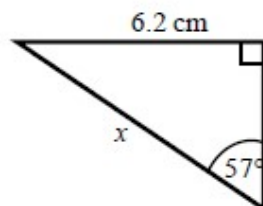
c



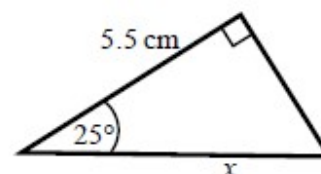
d



e

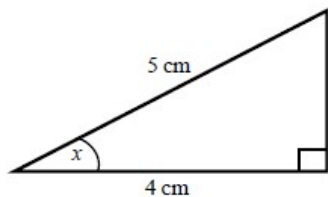


f

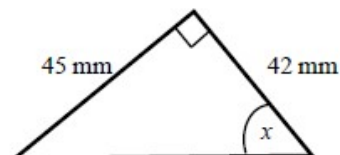


- 2 Calculate the size of angle  $x$  in each triangle.  
Give your answers correct to 1 decimal place.

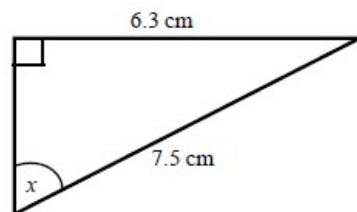
a



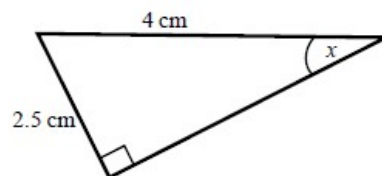
c



b



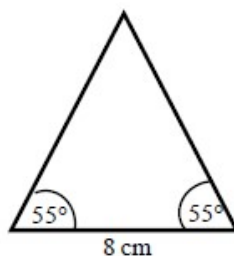
d



- 3 Work out the height of the isosceles triangle.  
Give your answer correct to 3 significant figures.

**Hint:**

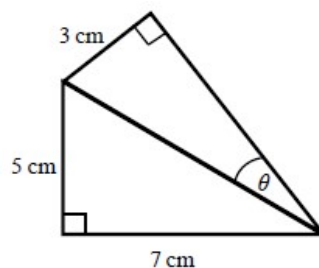
Split the triangle into two right-angled triangles.



- 4 Calculate the size of angle  $\theta$ .  
Give your answer correct to 1 decimal place.

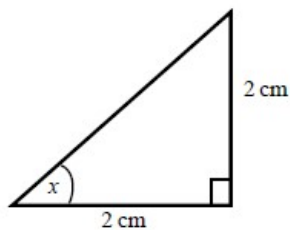
**Hint:**

First work out the length of the common side to both triangles, leaving your answer in surd form.

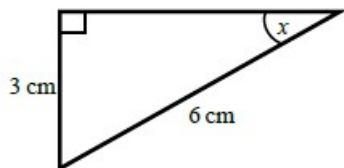


- 5 Find the exact value of  $x$  in each triangle.

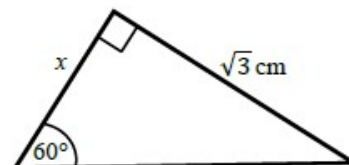
a



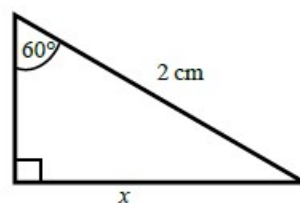
c



b



d



# The cosine rule

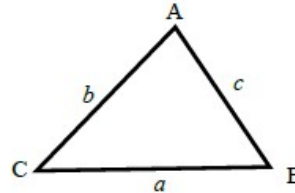
## A LEVEL LINKS

**Scheme of work:** 4a. Trigonometric ratios and graphs

**Textbook:** Pure Year 1, 9.1 The cosine rule

## Key points

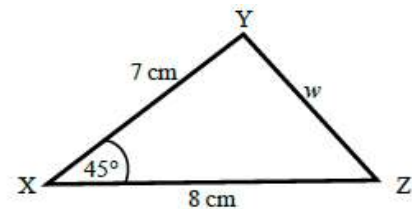
- $a$  is the side opposite angle  $A$ .
- $b$  is the side opposite angle  $B$ .
- $c$  is the side opposite angle  $C$ .

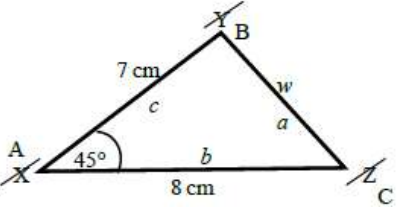


- You can use the cosine rule to find the length of a side when two sides and the included angle are given.
- To calculate an unknown side use the formula  $a^2 = b^2 + c^2 - 2bc \cos A$ .
- Alternatively, you can use the cosine rule to find an unknown angle if the lengths of all three sides are given.
- To calculate an unknown angle use the formula  $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$ .

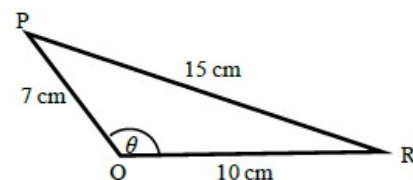
## Examples

**Example 4** Work out the length of side  $w$ .  
Give your answer correct to 3 significant figures.



 $a^2 = b^2 + c^2 - 2bc \cos A$ $w^2 = 8^2 + 7^2 - 2 \times 8 \times 7 \times \cos 45^\circ$ $w^2 = 33.804\,040\,51\dots$ $w = \sqrt{33.804\,040\,51}$ $w = 5.81 \text{ cm}$	<ol style="list-style-type: none"> <li>1 Always start by labelling the angles and sides.</li> <li>2 Write the cosine rule to find the side.</li> <li>3 Substitute the values <math>a</math>, <math>b</math> and <math>A</math> into the formula.</li> <li>4 Use a calculator to find <math>w^2</math> and then <math>w</math>.</li> <li>5 Round your final answer to 3 significant figures and write the units in your answer.</li> </ol>
---	---

**Example 5** Work out the size of angle  $\theta$ .  
Give your answer correct to 1 decimal place.

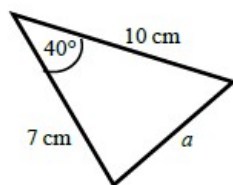


$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$ $\cos \theta = \frac{10^2 + 7^2 - 15^2}{2 \times 10 \times 7}$ $\cos \theta = \frac{-76}{140}$ $\theta = 122.878\ 349\dots$ $\theta = 122.9^\circ$	<ol style="list-style-type: none"> <li>1 Always start by labelling the angles and sides.</li> <li>2 Write the cosine rule to find the angle.</li> <li>3 Substitute the values <math>a</math>, <math>b</math> and <math>c</math> into the formula.</li> <li>4 Use <math>\cos^{-1}</math> to find the angle.</li> <li>5 Use your calculator to work out <math>\cos^{-1}(-76 \div 140)</math>.</li> <li>6 Round your answer to 1 decimal place and write the units in your answer.</li> </ol>
---	--

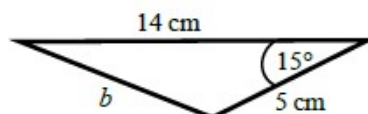
## Practice

**6** Work out the length of the unknown side in each triangle.  
Give your answers correct to 3 significant figures.

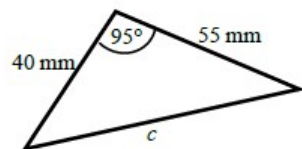
**a**



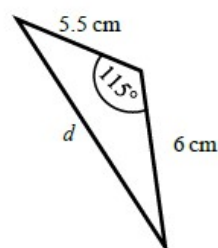
**b**



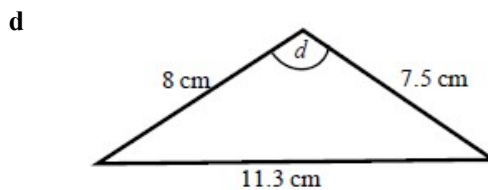
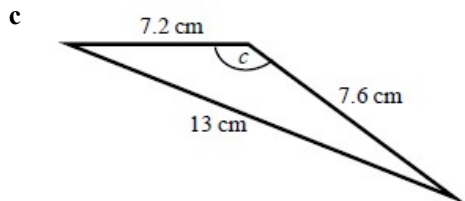
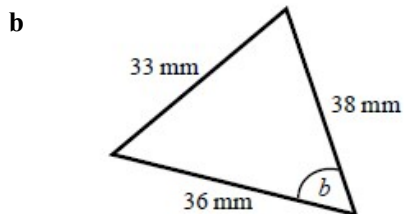
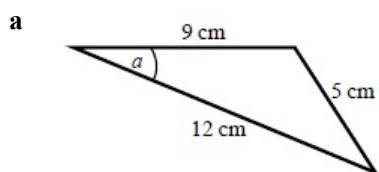
**c**



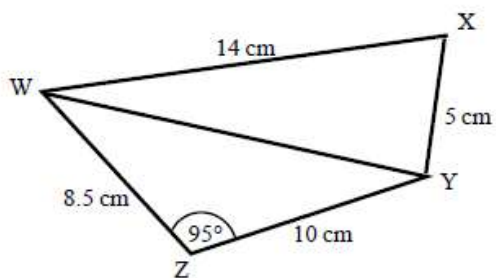
**d**



- 7 Calculate the angles labelled  $\theta$  in each triangle.  
Give your answer correct to 1 decimal place.



- 8 **a** Work out the length of WY.  
Give your answer correct to 3 significant figures.
- b** Work out the size of angle WXY.  
Give your answer correct to 1 decimal place.



# The sine rule

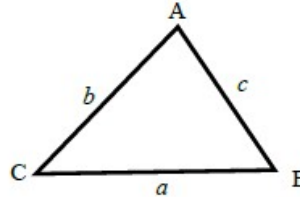
## A LEVEL LINKS

**Scheme of work:** 4a. Trigonometric ratios and graphs

**Textbook:** Pure Year 1, 9.2 The sine rule

## Key points

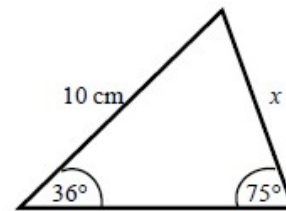
- $a$  is the side opposite angle  $A$ .
- $b$  is the side opposite angle  $B$ .
- $c$  is the side opposite angle  $C$ .

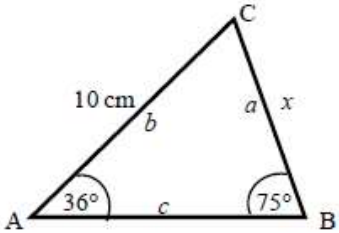


- You can use the sine rule to find the length of a side when its opposite angle and another opposite side and angle are given.
- To calculate an unknown side use the formula  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ .
- Alternatively, you can use the sine rule to find an unknown angle if the opposite side and another opposite side and angle are given.
- To calculate an unknown angle use the formula  $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ .

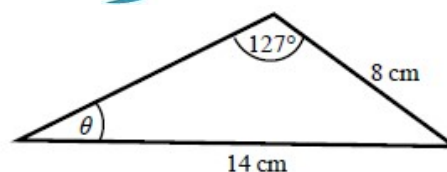
## Examples

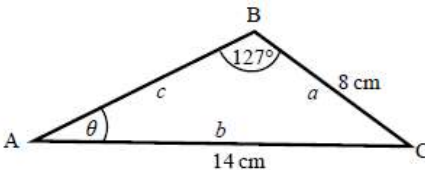
**Example 6** Work out the length of side  $x$ .  
Give your answer correct to 3 significant figures.



 $\frac{a}{\sin A} = \frac{b}{\sin B}$ $\frac{x}{\sin 36^\circ} = \frac{10}{\sin 75^\circ}$ $x = \frac{10 \times \sin 36^\circ}{\sin 75^\circ}$ $x = 6.09 \text{ cm}$	<ol style="list-style-type: none"> <li>1 Always start by labelling the angles and sides.</li> <li>2 Write the sine rule to find the side.</li> <li>3 Substitute the values <math>a</math>, <math>b</math>, <math>A</math> and <math>B</math> into the formula.</li> <li>4 Rearrange to make <math>x</math> the subject.</li> <li>5 Round your answer to 3 significant figures and write the units in your answer.</li> </ol>
--	--

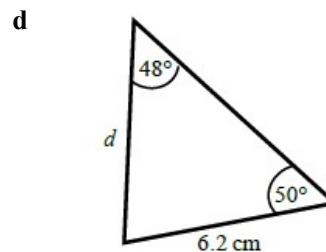
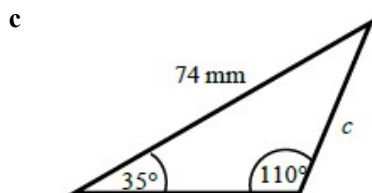
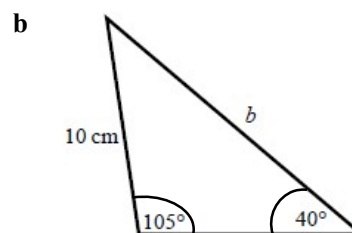
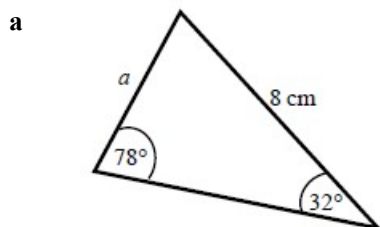
**Example 7** Work out the size of angle  $\theta$ .  
Give your answer correct to 1 decimal place.



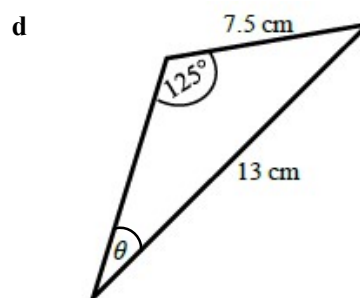
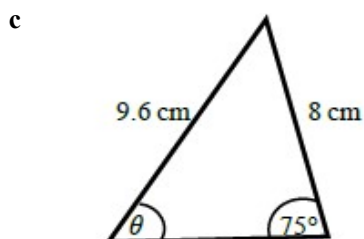
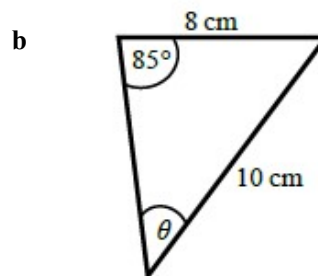
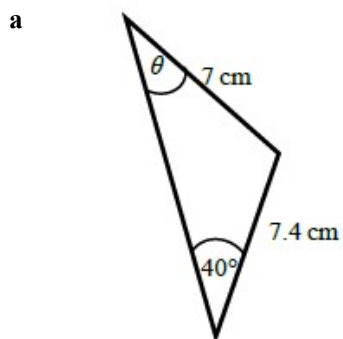
 $\frac{\sin A}{a} = \frac{\sin B}{b}$ $\frac{\sin \theta}{8} = \frac{\sin 127^\circ}{14}$ $\sin \theta = \frac{8 \times \sin 127^\circ}{14}$ $\theta = 27.2^\circ$	<ol style="list-style-type: none"> <li>1 Always start by labelling the angles and sides.</li> <li>2 Write the sine rule to find the angle.</li> <li>3 Substitute the values <math>a</math>, <math>b</math>, <math>A</math> and <math>B</math> into the formula.</li> <li>4 Rearrange to make <math>\sin \theta</math> the subject.</li> <li>5 Use <math>\sin^{-1}</math> to find the angle. Round your answer to 1 decimal place and write the units in your answer.</li> </ol>
--	---

## Practice

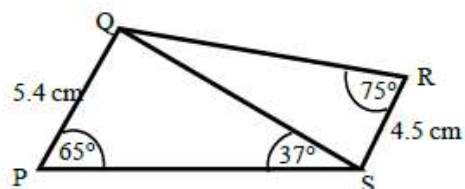
9 Find the length of the unknown side in each triangle.  
Give your answers correct to 3 significant figures.



- 10 Calculate the angles labelled  $\theta$  in each triangle.  
Give your answer correct to 1 decimal place.



- 11 a Work out the length of QS.  
Give your answer correct to 3 significant figures.
- b Work out the size of angle RQS.  
Give your answer correct to 1 decimal place.





# Areas of triangles

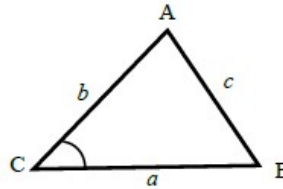
## A LEVEL LINKS

**Scheme of work:** 4a. Trigonometric ratios and graphs

**Textbook:** Pure Year 1, 9.3 Areas of triangles

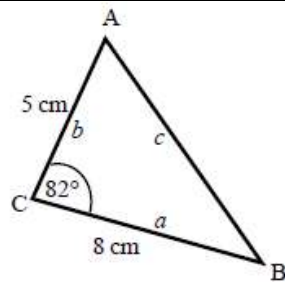
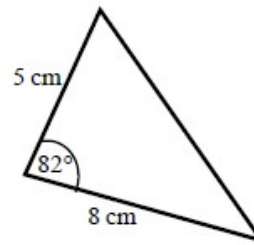
## Key points

- $a$  is the side opposite angle  $A$ .  
 $b$  is the side opposite angle  $B$ .  
 $c$  is the side opposite angle  $C$ .
- The area of the triangle is  $\frac{1}{2}ab \sin C$ .



## Examples

**Example 8** Find the area of the triangle.



$$\text{Area} = \frac{1}{2}ab \sin C$$

$$\text{Area} = \frac{1}{2} \times 8 \times 5 \times \sin 82^\circ$$

$$\text{Area} = 19.805361\dots$$

$$\text{Area} = 19.8 \text{ cm}^2$$

**1** Always start by labelling the sides and angles of the triangle.

**2** State the formula for the area of a triangle.

**3** Substitute the values of  $a$ ,  $b$  and  $C$  into the formula for the area of a triangle.

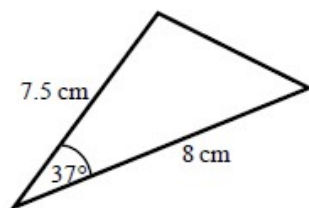
**4** Use a calculator to find the area.

**5** Round your answer to 3 significant figures and write the units in your answer.

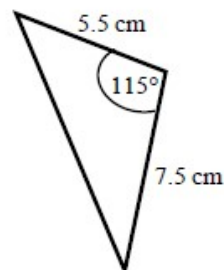
## Practice

- 12 Work out the area of each triangle.  
Give your answers correct to 3 significant figures.

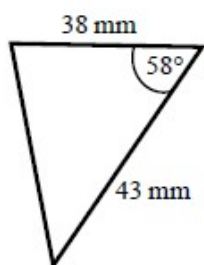
a



b



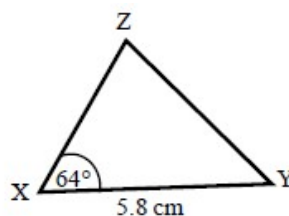
c



- 13 The area of triangle XYZ is  $13.3 \text{ cm}^2$ .  
Work out the length of XZ.

**Hint:**

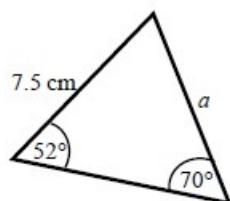
Rearrange the formula to make a side the subject.



## Extend

- 14 Find the size of each lettered angle or side.  
Give your answers correct to 3 significant figures.

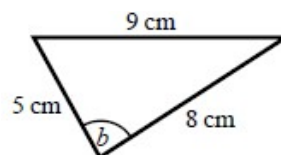
a



**Hint:**

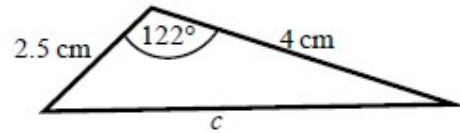
For each one, decide whether to use the cosine or sine rule.

b

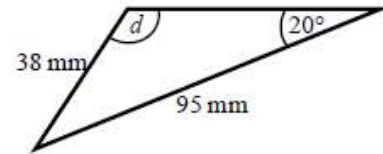




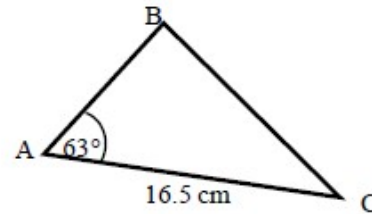
c



d



- 15 The area of triangle ABC is  $86.7 \text{ cm}^2$ .  
Work out the length of BC.  
Give your answer correct to 3 significant figures.



# Rearranging equations

## A LEVEL LINKS

**Scheme of work:** 6a. Definition, differentiating polynomials, second derivatives

**Textbook:** Pure Year 1, 12.1 Gradients of curves

## Key points

- To change the subject of a formula, get the terms containing the subject on one side and everything else on the other side.
- You may need to factorise the terms containing the new subject.

## Examples

**Example 1** Make  $t$  the subject of the formula  $v = u + at$ .

$v = u + at$ $v - u = at$ $t = \frac{v - u}{a}$	<ol style="list-style-type: none"> <li>1 Get the terms containing <math>t</math> on one side and everything else on the other side.</li> <li>2 Divide throughout by <math>a</math>.</li> </ol>
---	--

**Example 2** Make  $t$  the subject of the formula  $r = 2t - \pi t$ .

$r = 2t - \pi t$ $r = t(2 - \pi)$ $t = \frac{r}{2 - \pi}$	<ol style="list-style-type: none"> <li>1 All the terms containing <math>t</math> are already on one side and everything else is on the other side.</li> <li>2 Factorise as <math>t</math> is a common factor.</li> <li>3 Divide throughout by <math>2 - \pi</math>.</li> </ol>
---	--

**Example 3** Make  $t$  the subject of the formula  $\frac{t+r}{5} = \frac{3t}{2}$ .

$\frac{t+r}{5} = \frac{3t}{2}$ $2t + 2r = 15t$ $2r = 13t$ $t = \frac{2r}{13}$	<ol style="list-style-type: none"> <li>1 Remove the fractions first by multiplying throughout by 10.</li> <li>2 Get the terms containing <math>t</math> on one side and everything else on the other side and simplify.</li> <li>3 Divide throughout by 13.</li> </ol>
---	--

**Example 4** Make  $t$  the subject of the formula  $r = \frac{3t+5}{t-1}$ .

$r = \frac{3t+5}{t-1}$ $r(t-1) = 3t+5$ $rt-r = 3t+5$ $rt-3t = 5+r$ $t(r-3) = 5+r$ $t = \frac{5+r}{r-3}$	<ol style="list-style-type: none"> <li>1 Remove the fraction first by multiplying throughout by <math>t-1</math>.</li> <li>2 Expand the brackets.</li> <li>3 Get the terms containing <math>t</math> on one side and everything else on the other side.</li> <li>4 Factorise the LHS as <math>t</math> is a common factor.</li> <li>5 Divide throughout by <math>r-3</math>.</li> </ol>
---	---

## Practice

Change the subject of each formula to the letter given in the brackets.

- |   |                                   |                                   |
|---|-----------------------------------|-----------------------------------|
| 1 $C = \pi d$ [ $d$ ]                       | 2 $P = 2l + 2w$ [ $w$ ]           | 3 $D = \frac{S}{T}$ [ $T$ ]       |
| 4 $p = \frac{q-r}{t}$ [ $t$ ]               | 5 $u = at - \frac{1}{2}t$ [ $t$ ] | 6 $V = ax + 4x$ [ $x$ ]           |
| 7 $\frac{y-7x}{2} = \frac{7-2y}{3}$ [ $y$ ] | 8 $x = \frac{2a-1}{3-a}$ [ $a$ ]  | 9 $x = \frac{b-c}{d}$ [ $d$ ]     |
| 10 $h = \frac{7g-9}{2+g}$ [ $g$ ]           | 11 $e(9+x) = 2e+1$ [ $e$ ]        | 12 $y = \frac{2x+3}{4-x}$ [ $x$ ] |

13 Make  $r$  the subject of the following formulae.

**a**  $A = \pi r^2$      
**b**  $V = \frac{4}{3}\pi r^3$      
**c**  $P = \pi r + 2r$      
**d**  $V = \frac{2}{3}\pi r^2 h$

14 Make  $x$  the subject of the following formulae.

**a**  $\frac{xy}{z} = \frac{ab}{cd}$      
**b**  $\frac{4\pi cx}{d} = \frac{3z}{py^2}$

15 Make  $\sin B$  the subject of the formula  $\frac{a}{\sin A} = \frac{b}{\sin B}$

16 Make  $\cos B$  the subject of the formula  $b^2 = a^2 + c^2 - 2ac \cos B$ .

## Extend

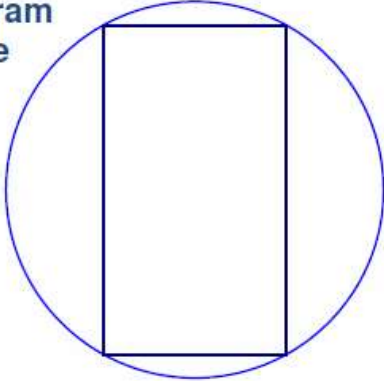
17 Make  $x$  the subject of the following equations.

**a**  $\frac{p}{q}(sx+t) = x-1$      
**b**  $\frac{p}{q}(ax+2y) = \frac{3p}{q^2}(x-y)$

# Additional Tasks

The following are a selection of problem-solving tasks from NRICH, STEP or UKMT for you to try, if you have time:

Question 1	Solve the system of equations: $ab = 1, bc = 2, cd = 3, de = 4, ea = 6$	Source: NRICH
Question 2	List all the real numbers $x$ such that $(x - 5)^{x^2 - 4} = 1$	
Question 3	Five numbers are arranged in order from least to greatest: $x, x^3, x^4, x^2, x^0$ Where does $-x^{-1}$ belong in the list above?	
Question 4	Prove that there are no positive integer solutions to the diophantine equation $x^2 - y^2 = 1$ .	
Question 5	Suppose that $a, b$ and $c$ are integers satisfying the equation $a^3 + 3b^3 = 9c^3$ .  Explain why $a$ must be divisible by 3. Show further that $b$ and $c$ must also be divisible by 3.  Deduce that the only integer solution is $a = b = c = 0$ .	Source: STEP
Question 6	Given the pair of simultaneous equations $ax + 2y = 1$ $2x + ay = b$  under what conditions on $a, b$ does there exist (i) a unique solution, (ii) no solution, (iii) an infinite number of solutions?	

<b>Question 7</b>	<p>The circle in the diagram has radius 6 cm. If the perimeter of the rectangle is 28 cm, what is its area?</p> 	<b>Source:</b> UKMT
<b>Question 8</b>	<p>(a) Express 87 as the product of two factors. (b) Express <math>b^2 - a^2</math> as the product of two factors. (c) Find all integer values of <math>x</math> for which <math>(x + 10)</math> and <math>(x + 97)</math> are both square numbers.</p>	

If you find that you have more time and want to do some more maths, the NRICH website has some great activities for you to try. These are just a few suggestions:

‘Funny factorisation’ <https://nrich.maths.org/funnyfactorisation>

‘Two and two’ <https://nrich.maths.org/twoandtwo>

‘Finding Factors’ <https://nrich.maths.org/7452>

‘Ben’s Game’ <https://nrich.maths.org/bensgame>

‘Wipeout’ <https://nrich.maths.org/wipeout>

‘Nine colours’ <https://nrich.maths.org/ninecolours>

‘Which Spinners’ <https://nrich.maths.org/6123>

‘Negatively triangular’ <https://nrich.maths.org/5871>

‘Generating Triples’ <https://nrich.maths.org/generatingtriples>

‘Pick’s Theorem’ <https://nrich.maths.org/pickstheorem>

## Answers - Expanding brackets and simplifying expressions

**1 a**  $6x - 3$

**b**  $-10pq - 8q^2$

**c**  $-3xy + 2y^2$

**2 a**  $21x + 35 + 12x - 48 = 33x - 13$

**b**  $40p - 16 - 12p - 27 = 28p - 43$

**c**  $27s + 9 - 30s + 50 = -3s + 59 = 59 - 3s$

**d**  $8x - 6 - 3x - 5 = 5x - 11$

**3 a**  $12x^2 + 24x$

**b**  $20k^3 - 48k$

**c**  $10h - 12h^3 - 22h^2$

**d**  $21s^2 - 21s^3 - 6s$

**4 a**  $-y^2 - 4$

**b**  $5x^2 - 11x$

**c**  $2p - 7p^2$

**d**  $6b^2$

**5**  $y - 4$

$$\mathbf{6} \quad \mathbf{a} \quad -1 - 2m$$

**b**  $5p^3 + 12p^2 + 27p$

7  $7x(3x - 5) = 21x^2 - 35x$

**8 a**  $x^2 + 9x + 20$

**b**  $x^2 + 10x + 21$

**c**  $x^2 + 5x - 14$

**d**  $x^2 - 25$

**e**  $2x^2 + x - 3$

**f**  $6x^2 - x - 2$

$$\mathbf{g} \quad 10x^2 - 31x + 15$$

### h $12x^2 + 13x - 14$

**i**  $18x^2 + 39xy + 20y^2$

**j**  $x^2 + 10x + 25$

**k**  $4x^2 - 28x + 49$

$$\mathbf{1} \quad 16x^2 - 24xy + 9y^2$$

**9**  $2x^2 - 2x + 25$

**10 a**  $x^2 - 1 - \frac{2}{x^2}$

**b**  $x^2 + 2 + \frac{1}{x^2}$



**Answers - Surds and rationalising the denominator**

1 a  $3\sqrt{5}$   
c  $4\sqrt{3}$   
e  $10\sqrt{3}$   
g  $6\sqrt{2}$

b  $5\sqrt{5}$   
d  $5\sqrt{7}$   
f  $2\sqrt{7}$   
h  $9\sqrt{2}$

2 a  $15\sqrt{2}$   
c  $3\sqrt{2}$   
e  $6\sqrt{7}$

b  $\sqrt{5}$   
d  $\sqrt{3}$   
f  $5\sqrt{3}$

3 a  $-1$   
c  $10\sqrt{5} - 7$

b  $9 - \sqrt{3}$   
d  $26 - 4\sqrt{2}$

4 a  $\frac{\sqrt{5}}{5}$   
c  $\frac{2\sqrt{7}}{7}$   
e  $\sqrt{2}$   
g  $\frac{\sqrt{3}}{3}$

b  $\frac{\sqrt{11}}{11}$   
d  $\frac{\sqrt{2}}{2}$   
f  $\sqrt{5}$   
h  $\frac{1}{3}$

5 a  $\frac{3+\sqrt{5}}{4}$

b  $\frac{2(4-\sqrt{3})}{13}$

c  $\frac{6(5+\sqrt{2})}{23}$

6  $x - y$

7 a  $3 + 2\sqrt{2}$

b  $\frac{\sqrt{x} + \sqrt{y}}{x - y}$

## Answers - Rules of indices

**1 a** 1

**b** 1

**c** 1

**d** 1

**2 a** 7

**b** 4

**c** 5

**d** 2

**3 a** 125

**b** 32

**c** 343

**d** 8

**4 a**  $\frac{1}{25}$

**b**  $\frac{1}{64}$

**c**  $\frac{1}{32}$

**d**  $\frac{1}{36}$

**5 a**  $\frac{3x^3}{2}$

**b**  $5x^2$

**c**  $3x$

**d**  $\frac{y}{2x^2}$

**e**  $y^{\frac{1}{2}}$

**f**  $c^{-3}$

**g**  $2x^6$

**h**  $x$

**6 a**  $\frac{1}{2}$

**b**  $\frac{1}{9}$

**c**  $\frac{8}{3}$

**d**  $\frac{1}{4}$

**e**  $\frac{4}{3}$

**f**  $\frac{16}{9}$

**7 a**  $x^{-1}$

**b**  $x^{-7}$

**c**  $x^{\frac{1}{4}}$

**d**  $x^{\frac{2}{5}}$

**e**  $x^{-\frac{1}{3}}$

**f**  $x^{\frac{2}{3}}$

**8 a**  $\frac{1}{x^3}$

**b** 1

**c**  $\sqrt[5]{x}$

**d**  $\sqrt[5]{x^2}$

**e**  $\frac{1}{\sqrt{x}}$

**f**  $\frac{1}{\sqrt[4]{x^3}}$

**9 a**  $5x^{\frac{1}{2}}$

**b**  $2x^{-3}$

**c**  $\frac{1}{3}x^{-4}$

**d**  $2x^{-\frac{1}{2}}$

**e**  $4x^{-\frac{1}{3}}$

**f**  $3x^0$

**10 a**  $x^3 + x^{-2}$

**b**  $x^3 + x$

**c**  $x^{-2} + x^{-7}$

## Answers - Factorising expressions

- |          |                                    |                                  |
|----------|------------------------------------|----------------------------------|
| <b>1</b> | <b>a</b> $2x^3y^3(3x - 5y)$        | <b>b</b> $7a^3b^2(3b^3 + 5a^2)$  |
|          | <b>c</b> $5x^2y^2(5 - 2x + 3y)$    |                                  |
| <b>2</b> | <b>a</b> $(x + 3)(x + 4)$          | <b>b</b> $(x + 7)(x - 2)$        |
|          | <b>c</b> $(x - 5)(x - 6)$          | <b>d</b> $(x - 8)(x + 3)$        |
|          | <b>e</b> $(x - 9)(x + 2)$          | <b>f</b> $(x + 5)(x - 4)$        |
|          | <b>g</b> $(x - 8)(x + 5)$          | <b>h</b> $(x + 7)(x - 4)$        |
| <b>3</b> | <b>a</b> $(6x - 7y)(6x + 7y)$      | <b>b</b> $(2x - 9y)(2x + 9y)$    |
|          | <b>c</b> $2(3a - 10bc)(3a + 10bc)$ |                                  |
| <b>4</b> | <b>a</b> $(x - 1)(2x + 3)$         | <b>b</b> $(3x + 1)(2x + 5)$      |
|          | <b>c</b> $(2x + 1)(x + 3)$         | <b>d</b> $(3x - 1)(3x - 4)$      |
|          | <b>e</b> $(5x + 3)(2x + 3)$        | <b>f</b> $2(3x - 2)(2x - 5)$     |
| <b>5</b> | <b>a</b> $\frac{2(x + 2)}{x - 1}$  | <b>b</b> $\frac{x}{x - 1}$       |
|          | <b>c</b> $\frac{x + 2}{x}$         | <b>d</b> $\frac{x}{x + 5}$       |
|          | <b>e</b> $\frac{x + 3}{x}$         | <b>f</b> $\frac{x}{x - 5}$       |
| <b>6</b> | <b>a</b> $\frac{3x + 4}{x + 7}$    | <b>b</b> $\frac{2x + 3}{3x - 2}$ |
|          | <b>c</b> $\frac{2 - 5x}{2x - 3}$   | <b>d</b> $\frac{3x + 1}{x + 4}$  |
| <b>7</b> | $(x + 5)$                          |                                  |
| <b>8</b> | $\frac{4(x + 2)}{x - 2}$           |                                  |

**Answers - Completing the square**

**1 a**  $(x+2)^2 - 1$

**b**  $(x-5)^2 - 28$

**c**  $(x-4)^2 - 16$

**d**  $(x+3)^2 - 9$

**e**  $(x-1)^2 + 6$

**f**  $\left(x + \frac{3}{2}\right)^2 - \frac{17}{4}$

**2 a**  $2(x-2)^2 - 24$

**b**  $4(x-1)^2 - 20$

**c**  $3(x+2)^2 - 21$

**d**  $2\left(x + \frac{3}{2}\right)^2 - \frac{25}{2}$

**3 a**  $2\left(x + \frac{3}{4}\right)^2 + \frac{39}{8}$

**b**  $3\left(x - \frac{1}{3}\right)^2 - \frac{1}{3}$

**c**  $5\left(x + \frac{3}{10}\right)^2 - \frac{9}{20}$

**d**  $3\left(x + \frac{5}{6}\right)^2 + \frac{11}{12}$

**4**  $(5x+3)^2 + 3$

**Answers - Solving linear simultaneous equations**

**1**  $x = 1, y = 4$

**2**  $x = 3, y = -2$

**3**  $x = 2, y = -5$

**4**  $x = 3, y = -\frac{1}{2}$

**5**  $x = 6, y = -1$

**6**  $x = -2, y = 5$

**7**  $x = 9, y = 5$

**8**  $x = -2, y = -7$

**9**  $x = \frac{1}{2}, y = 3\frac{1}{2}$

**10**  $x = \frac{1}{2}, y = 3$

**11**  $x = -4, y = 5$

**12**  $x = -2, y = -5$

**13**  $x = \frac{1}{4}, y = 1\frac{3}{4}$

**14**  $x = -2, y = 2\frac{1}{2}$

**15**  $x = -2\frac{1}{2}, y = 5\frac{1}{2}$

**Answers - Pythagoras' theorem**

- 1**   **a**   10.3 cm                      **b**   7.07 cm  
         **c**   58.6 mm                    **d**   8.94 cm
- 2**   **a**    $4\sqrt{3}$  cm                    **b**    $2\sqrt{21}$  cm  
         **c**    $8\sqrt{17}$  mm                **d**    $18\sqrt{5}$  mm
- 3**   **a**    $18\sqrt{13}$  mm                **b**    $2\sqrt{145}$  mm  
         **c**    $42\sqrt{2}$  mm                **d**    $6\sqrt{89}$  mm
- 4**   95.3 mm
- 5**   64.0 km
- 6**    $3\sqrt{5}$  units
- 7**    $4\sqrt{3}$  cm

**Answers - Trigonometry**

- |          |          |         |          |         |          |         |  |
|----------|----------|---------|----------|---------|----------|---------|--|
| <b>1</b> | <b>a</b> | 6.49 cm | <b>b</b> | 6.93 cm | <b>c</b> | 2.80 cm |  |
|          | <b>d</b> | 74.3 mm | <b>e</b> | 7.39 cm | <b>f</b> | 6.07 cm |  |
- 
- |          |          |       |          |       |          |       |          |       |
|----------|----------|-------|----------|-------|----------|-------|----------|-------|
| <b>2</b> | <b>a</b> | 36.9° | <b>b</b> | 57.1° | <b>c</b> | 47.0° | <b>d</b> | 38.7° |
|----------|----------|-------|----------|-------|----------|-------|----------|-------|
- 
- 3** 5.71 cm
- 
- 4** 20.4°
- 
- |          |          |     |          |      |          |     |          |               |
|----------|----------|-----|----------|------|----------|-----|----------|---------------|
| <b>5</b> | <b>a</b> | 45° | <b>b</b> | 1 cm | <b>c</b> | 30° | <b>d</b> | $\sqrt{3}$ cm |
|----------|----------|-----|----------|------|----------|-----|----------|---------------|
- 
- |          |          |         |          |         |          |         |          |         |
|----------|----------|---------|----------|---------|----------|---------|----------|---------|
| <b>6</b> | <b>a</b> | 6.46 cm | <b>b</b> | 9.26 cm | <b>c</b> | 70.8 mm | <b>d</b> | 9.70 cm |
|----------|----------|---------|----------|---------|----------|---------|----------|---------|
- 
- |          |          |       |          |       |          |        |          |       |
|----------|----------|-------|----------|-------|----------|--------|----------|-------|
| <b>7</b> | <b>a</b> | 22.2° | <b>b</b> | 52.9° | <b>c</b> | 122.9° | <b>d</b> | 93.6° |
|----------|----------|-------|----------|-------|----------|--------|----------|-------|
- 
- |          |          |         |          |       |  |  |  |  |
|----------|----------|---------|----------|-------|--|--|--|--|
| <b>8</b> | <b>a</b> | 13.7 cm | <b>b</b> | 76.0° |  |  |  |  |
|----------|----------|---------|----------|-------|--|--|--|--|
- 
- |          |          |         |          |         |          |         |          |         |
|----------|----------|---------|----------|---------|----------|---------|----------|---------|
| <b>9</b> | <b>a</b> | 4.33 cm | <b>b</b> | 15.0 cm | <b>c</b> | 45.2 mm | <b>d</b> | 6.39 cm |
|----------|----------|---------|----------|---------|----------|---------|----------|---------|
- 
- |           |          |       |          |       |          |       |          |       |
|-----------|----------|-------|----------|-------|----------|-------|----------|-------|
| <b>10</b> | <b>a</b> | 42.8° | <b>b</b> | 52.8° | <b>c</b> | 53.6° | <b>d</b> | 28.2° |
|-----------|----------|-------|----------|-------|----------|-------|----------|-------|
- 
- |           |          |         |          |       |  |  |  |  |
|-----------|----------|---------|----------|-------|--|--|--|--|
| <b>11</b> | <b>a</b> | 8.13 cm | <b>b</b> | 32.3° |  |  |  |  |
|-----------|----------|---------|----------|-------|--|--|--|--|
- 
- |           |          |                      |          |                      |          |                     |  |  |
|-----------|----------|----------------------|----------|----------------------|----------|---------------------|--|--|
| <b>12</b> | <b>a</b> | 18.1 cm <sup>2</sup> | <b>b</b> | 18.7 cm <sup>2</sup> | <b>c</b> | 693 mm <sup>2</sup> |  |  |
|-----------|----------|----------------------|----------|----------------------|----------|---------------------|--|--|
- 
- 13** 5.10 cm
- 
- |           |          |         |          |       |          |         |          |       |
|-----------|----------|---------|----------|-------|----------|---------|----------|-------|
| <b>14</b> | <b>a</b> | 6.29 cm | <b>b</b> | 84.3° | <b>c</b> | 5.73 cm | <b>d</b> | 58.8° |
|-----------|----------|---------|----------|-------|----------|---------|----------|-------|
- 
- 15** 15.3 cm

## Answers - Rearranging equations

1  $d = \frac{C}{\pi}$

2  $w = \frac{P-2l}{2}$

3  $T = \frac{S}{D}$

4  $t = \frac{q-r}{p}$

5  $t = \frac{2u}{2a-1}$

6  $x = \frac{V}{a+4}$

7  $y = 2 + 3x$

8  $a = \frac{3x+1}{x+2}$

9  $d = \frac{b-c}{x}$

10  $g = \frac{2h+9}{7-h}$

11  $e = \frac{1}{x+7}$

12  $x = \frac{4y-3}{2+y}$

13 a  $r = \sqrt{\frac{A}{\pi}}$

b  $r = \sqrt[3]{\frac{3V}{4\pi}}$

c  $r = \frac{P}{\pi+2}$

d  $r = \sqrt{\frac{3V}{2\pi h}}$

14 a  $x = \frac{abz}{cdy}$

b  $x = \frac{3dz}{4\pi cpy^2}$

15  $\sin B = \frac{b \sin A}{a}$

16  $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$

17 a  $x = \frac{q+pt}{q-ps}$

b  $x = \frac{3py+2pqy}{3p-apq} = \frac{y(3+2q)}{3-aq}$





## Solutions to Problem-solving Tasks

<p><b>Question 1</b></p>	$\left. \begin{array}{l} ab = 1 \\ bc = 2 \\ cd = 3 \\ de = 4 \\ ea = 6 \end{array} \right\} \begin{array}{l} \text{multiply all} \\ \text{together} \end{array}$ $\Rightarrow a^2 b^2 c^2 d^2 e^2 = 144$ $\Rightarrow abcde = \pm 12$ <p>Suppose <math>abcde = 12</math>. Then we have by substitution from above</p> $\begin{aligned} 3e &= 12 \\ \Rightarrow e &= 4 \\ \Rightarrow d &= 1 \\ \Rightarrow a &= \frac{3}{2} \\ \Rightarrow b &= \frac{2}{3} \\ \Rightarrow c &= 3 \end{aligned}$ <p>If on the other hand <math>abcde = -12</math></p> $\begin{aligned} 3e &= -12 \\ \Rightarrow e &= -4 \\ \Rightarrow d &= -1 \\ \Rightarrow a &= -\frac{3}{2} \\ \Rightarrow b &= -\frac{2}{3} \\ \Rightarrow c &= -3 \end{aligned}$ <p>So just two solutions listed.</p>
<p><b>Question 2</b></p>	$(x-5)^{x^2-4} = 1$ $\Rightarrow 0 = x^2 - 4 \quad (0 \neq x-5) \text{ or } 1 = x-5$ $\Rightarrow x = \pm 2 \quad \left( \begin{array}{l} x^2+5 \\ \text{No OK} \end{array} \right) \quad \Rightarrow x = 6$ $x = \pm 2 \text{ or } 6$
<p><b>Question 3</b></p>	<p>Assume <math>x \neq 0</math> (so <math>x^0</math> is defined). Then</p> $x^2 > 0, x^4 > 0$ <p>and <math>x^2 &gt; x^4</math></p> $\Rightarrow  x  < 1$ <p>Then <math>x^3 &lt; x^4</math></p> $\Rightarrow x < 0$ <p>so <math>-1 &lt; x &lt; 0</math>.</p> <p>Hence <math>-\frac{1}{x} &gt; 0</math></p> <p>and <math>-\frac{1}{x} &gt; 1 = x^0</math></p> <p>so <math>-x^{-1}</math> belongs last.</p>



### Question 4

$$x^2 - y^2 = 1$$

$$\Rightarrow (x-y)(x+y) = 1$$

$$\Rightarrow \begin{cases} x-y = 1 \\ x+y = 1 \end{cases} \text{ or } \begin{cases} x-y = -1 \\ x+y = -1 \end{cases}$$

$$\Rightarrow (x=1, y=0) \text{ or } (x=-1, y=0)$$

Neither of these consists of exclusively positive integers.

### Question 5

$$a^3 + 3b^3 = 9c^3$$

$$\Rightarrow a^3 = 9c^3 - 3b^3$$

$$= 3(3c^3 - b^3)$$

hence  $3|a$  (and possibly  $a=0$ ).

let  $a = 3d$

$$\Rightarrow 27d^3 = 3(3c^3 - b^3)$$

$$\Rightarrow 9d^3 = 3c^3 - b^3$$

$$\Rightarrow b^3 = 3c^3 - 9d^3$$

$$= 3(c^3 - 3d^3)$$

$$\Rightarrow 3|b$$

let  $b = 3e$

$$27d^3 = 3(c^3 - 3e^3)$$

$$\Rightarrow 9d^3 = c^3 - 3e^3$$

$$\Rightarrow c^3 = 3d^3 + 9e^3$$

$$= 3(d^3 + 3e^3)$$

$$\Rightarrow 3|c.$$

So all of  $a, b, c$  are multiples of 3.

Given

$$a^3 + 3b^3 = 9c^3$$

let  $3^n$  be the highest power of 3 which divides all three.

So  $a = 3^n p, b = 3^n q, c = 3^n r$  where  $p, q, r$  are integers. Then

$$p^3 + 3q^3 = 9r^3$$

But then all of  $p, q, r$  are divisible by 3 - a contradiction.

So there is no highest power, and so  $a=b=c=0$  is the only solution.

### Question 6

$$ax + 2y = 1$$

$$2x + ay = b$$

$$\begin{vmatrix} a & 2 \\ 2 & a \end{vmatrix} = a^2 - 4$$

so if  $a \neq \pm 2$ , there is a unique solution for  $(x, y)$ .

If  $a = 2$ ,

$$2x + 2y = 1$$

$$2x + 2y = b$$

so there is an infinite number of solutions if  $b = 1$ .

If  $a = -2$ ,

$$-2x + 2y = 1$$

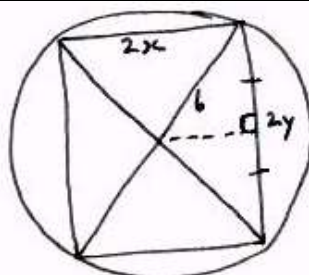
$$2x - 2y = b$$

so there is an infinite number of solutions if  $b = -1$ .

$a$	$b$	$(x, y)$
$\neq 2$		unique soln exists
2	1	infinite no of solns
2	$\neq 1$	no solution
-2	-1	infinite no of solns
	$\neq -1$	no solutions



### Question 7



By Pythagoras'

$$x^2 + y^2 = 36$$

$$\text{and } x + y = 7 \quad (*)$$

$$\text{Area} = 4xy$$

$$(*) \Rightarrow x^2 + 2xy + y^2 = 49$$

$$\text{so } 2xy = 13$$

$$\Rightarrow 4xy = 26$$

$$\text{Area is } 26 \text{ cm}^2$$

### Question 8

a)  $87 = 3 \times 29$ .

$$b^2 - a^2 = (b - a)(b + a)$$

c) suppose

$$m^2 = x + 10$$

$$n^2 = x + 97$$

$$\Rightarrow n^2 - m^2 = 87$$

$$\Rightarrow (n - m)(n + m) = 3 \times 29$$

$$\Rightarrow \begin{cases} n - m = 3 \\ n + m = 29 \end{cases} \text{ or } \begin{cases} n + m = 3 \\ n - m = 29 \end{cases}$$

since 3, 29 are primes.

$$\Rightarrow (n = 16, m = 13) \text{ or } (n = 16, m = -13)$$

$$\Rightarrow x = 159 \text{ or } x = 159$$

$$\text{i.e. just } x = 159$$